

# Übungsbeispiel $\alpha$ - $\beta$ -Algorithmus

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# Übungsbeispiel $\alpha$ - $\beta$ -Algorithmus

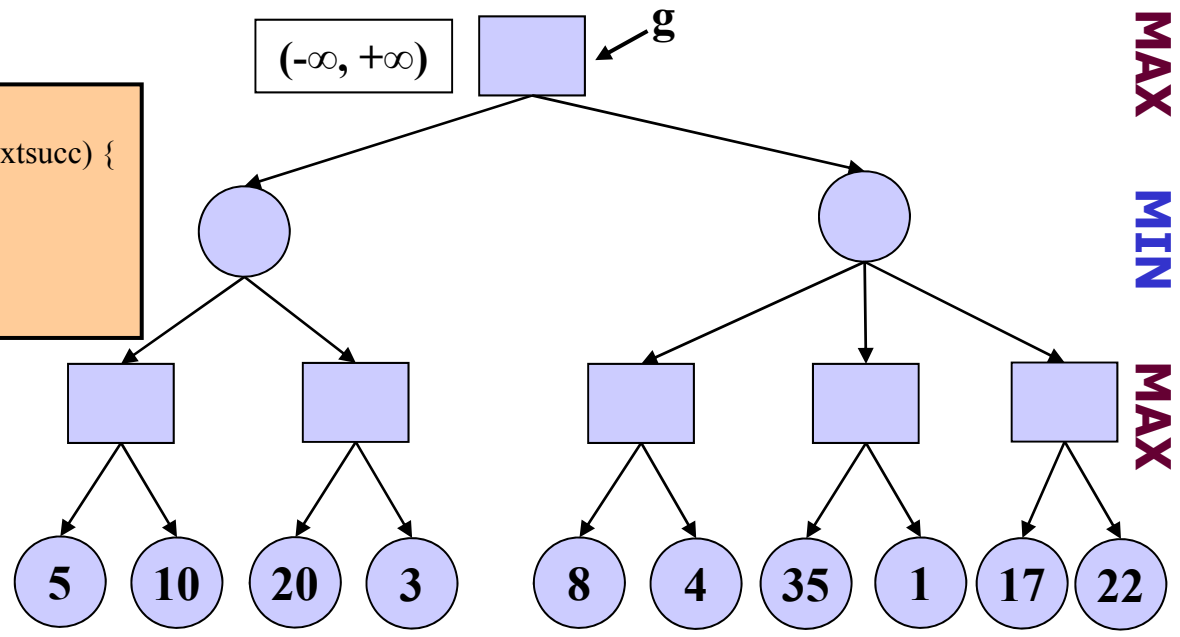
- Illustriert auf 75 Folien Schritt für Schritt den Ablauf des  $\alpha$ - $\beta$ -Algorithmus

```
int maxValue(Gamestate g, int  $\alpha$ , int  $\beta$ ) {
    if cutofftest(g) return eval(g);
    for (GameState s=g.firstsucc; s!=g.lastsucc; s=s.nextsucc) {
         $\alpha$  = max( $\alpha$ , minValue(s,  $\alpha$ ,  $\beta$  ));
        if ( $\alpha$  >=  $\beta$ ) break; //  $\beta$ -Schnitt
    }
    return  $\alpha$ ;
}

int minValue(Gamestate g, int  $\alpha$ , int  $\beta$ ) {
    if cutofftest(g) return eval(g);
    for (GameState s=g.firstsucc; s!=g.lastsucc; s=s.nextsucc) {
         $\beta$  = min( $\beta$ , maxValue(s,  $\alpha$ ,  $\beta$  ));
        if ( $\beta$  <=  $\alpha$ ) break; //  $\alpha$ -Schnitt
    }
    return  $\beta$ ;
}
```

**maxValue(g, -∞, +∞)**

```
cutofftest(g) → false  
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {  
    α = max(-∞, minValue(s, -∞, +∞));  
    if (α ≥ +∞) break;  
}  
return α;
```

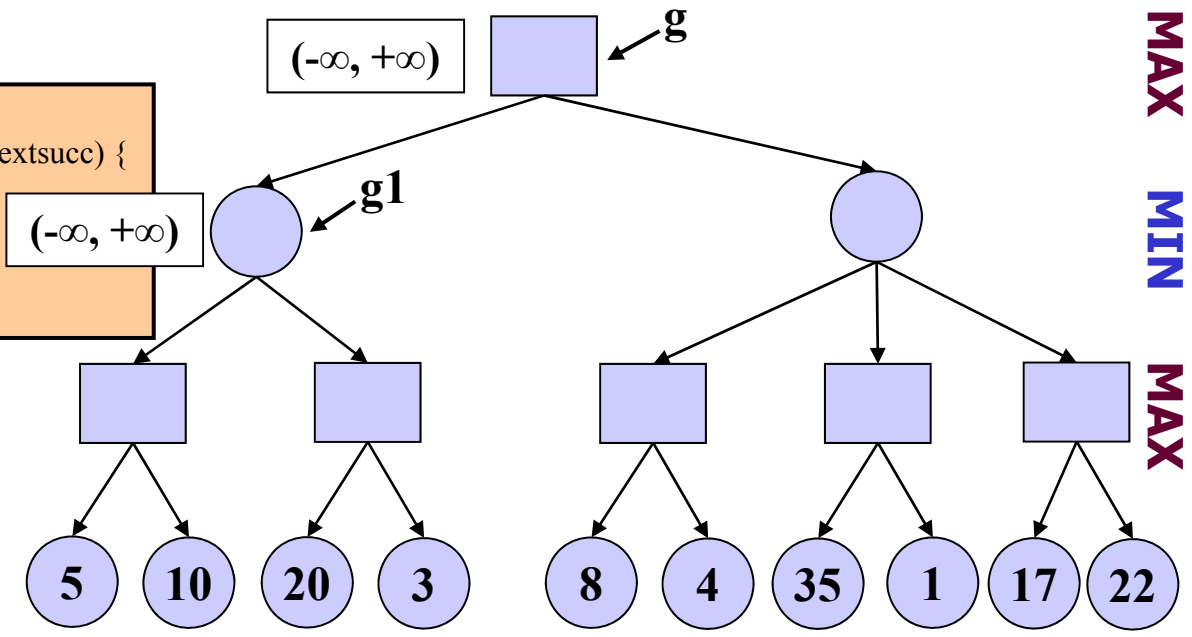


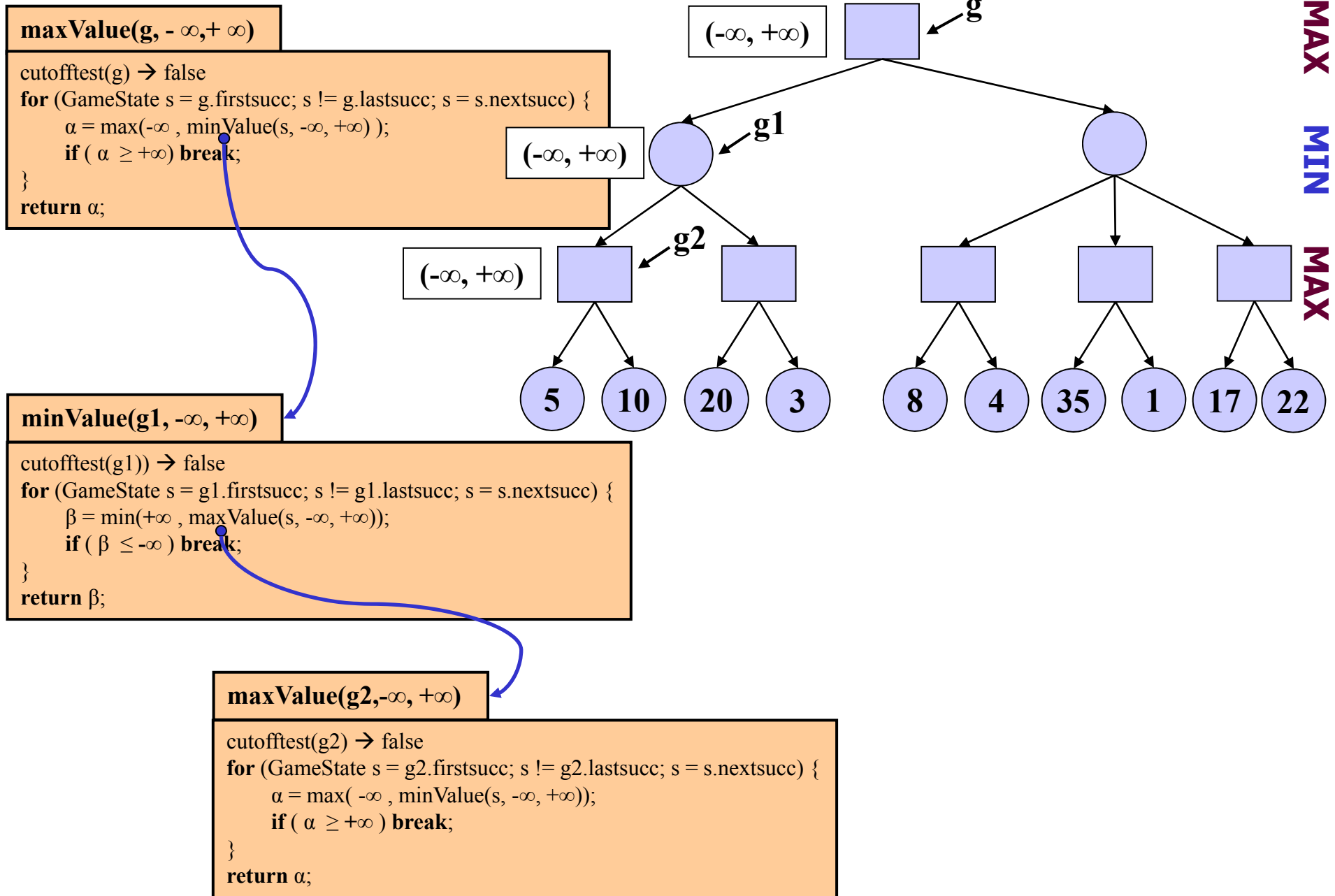
```
maxValue(g, -∞, +∞)
```

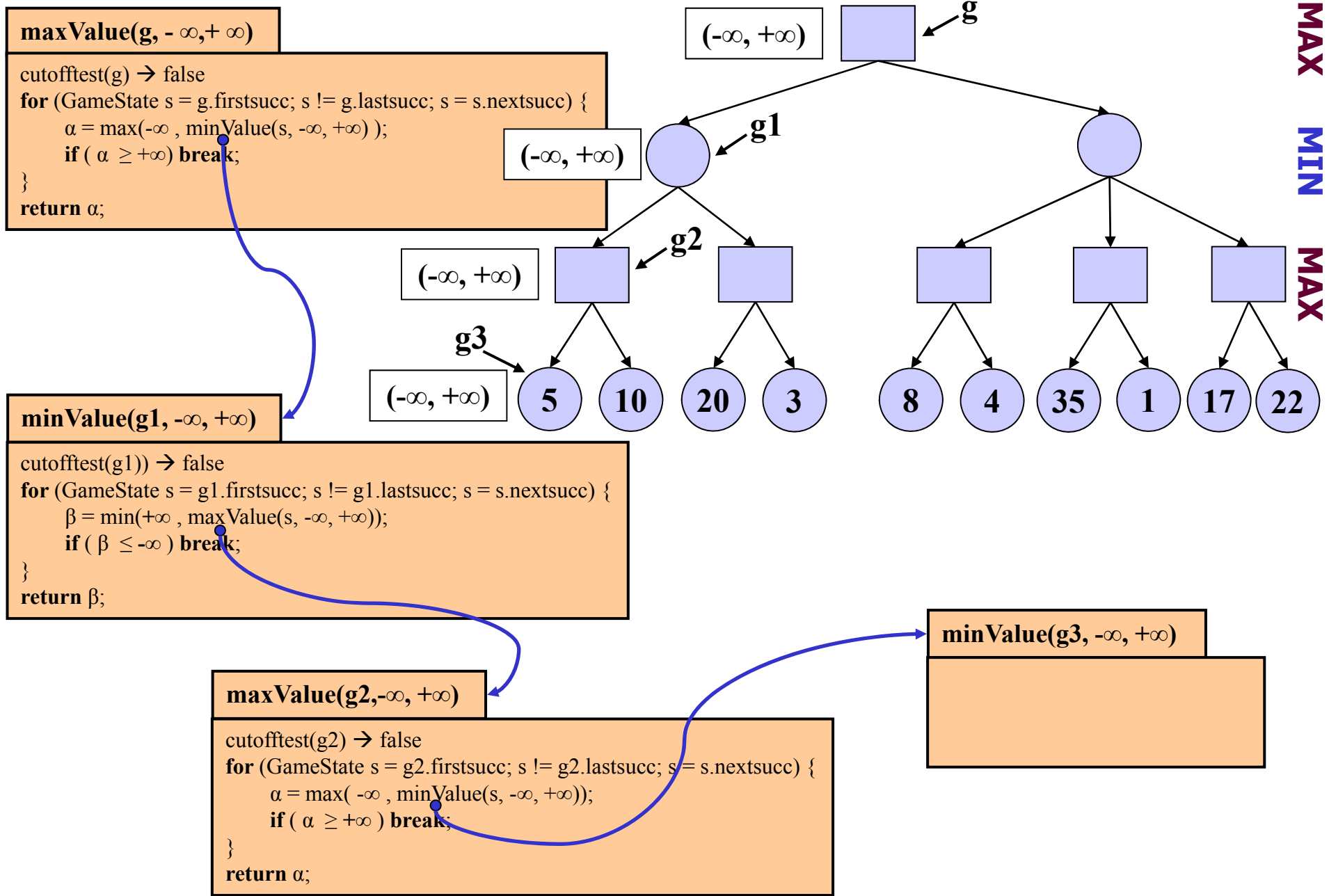
```
cutofftest(g) → false  
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {  
    α = max(-∞, minValue(s, -∞, +∞));  
    if (α ≥ +∞) break;  
}  
return α;
```

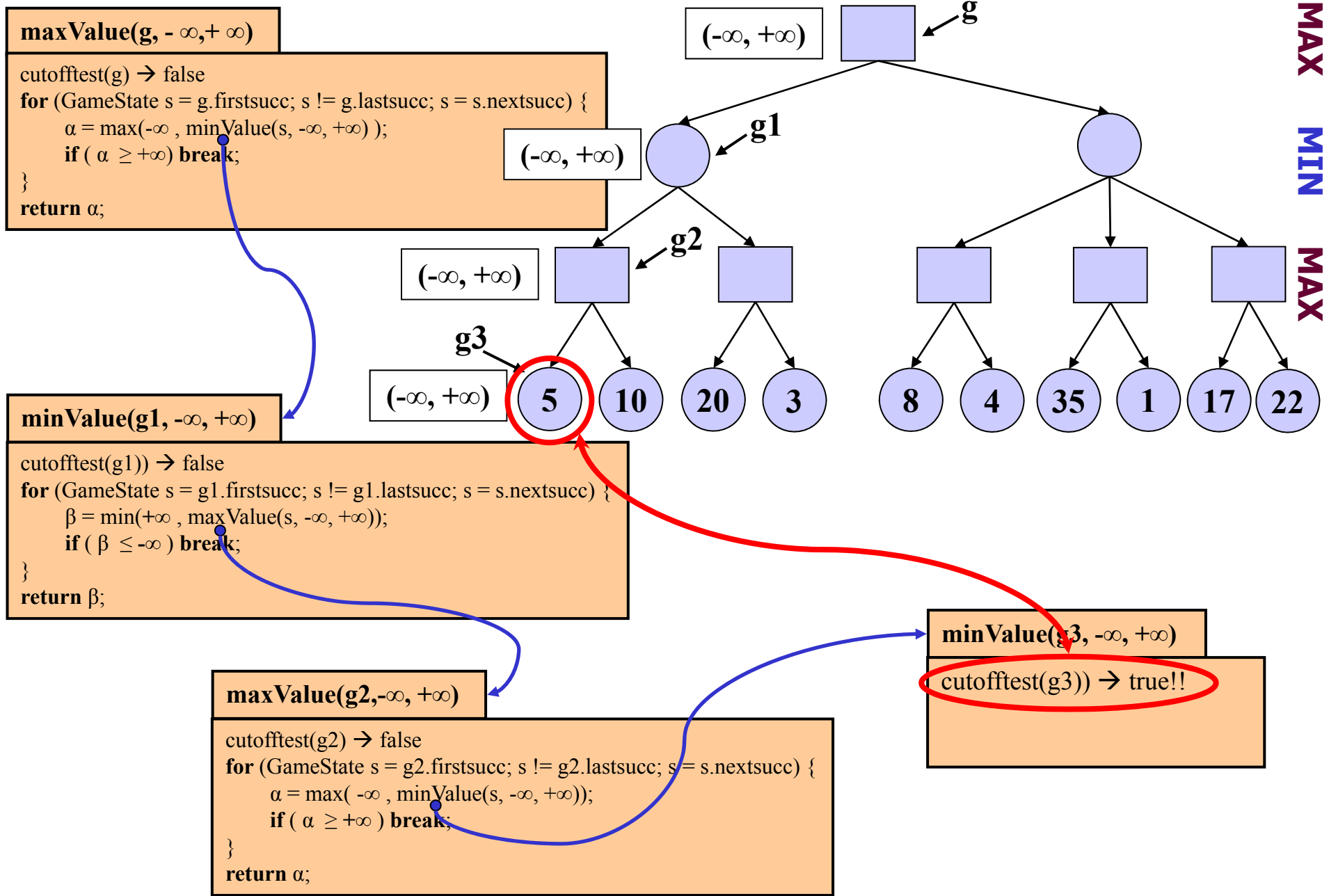
```
minValue(g1, -∞, +∞)
```

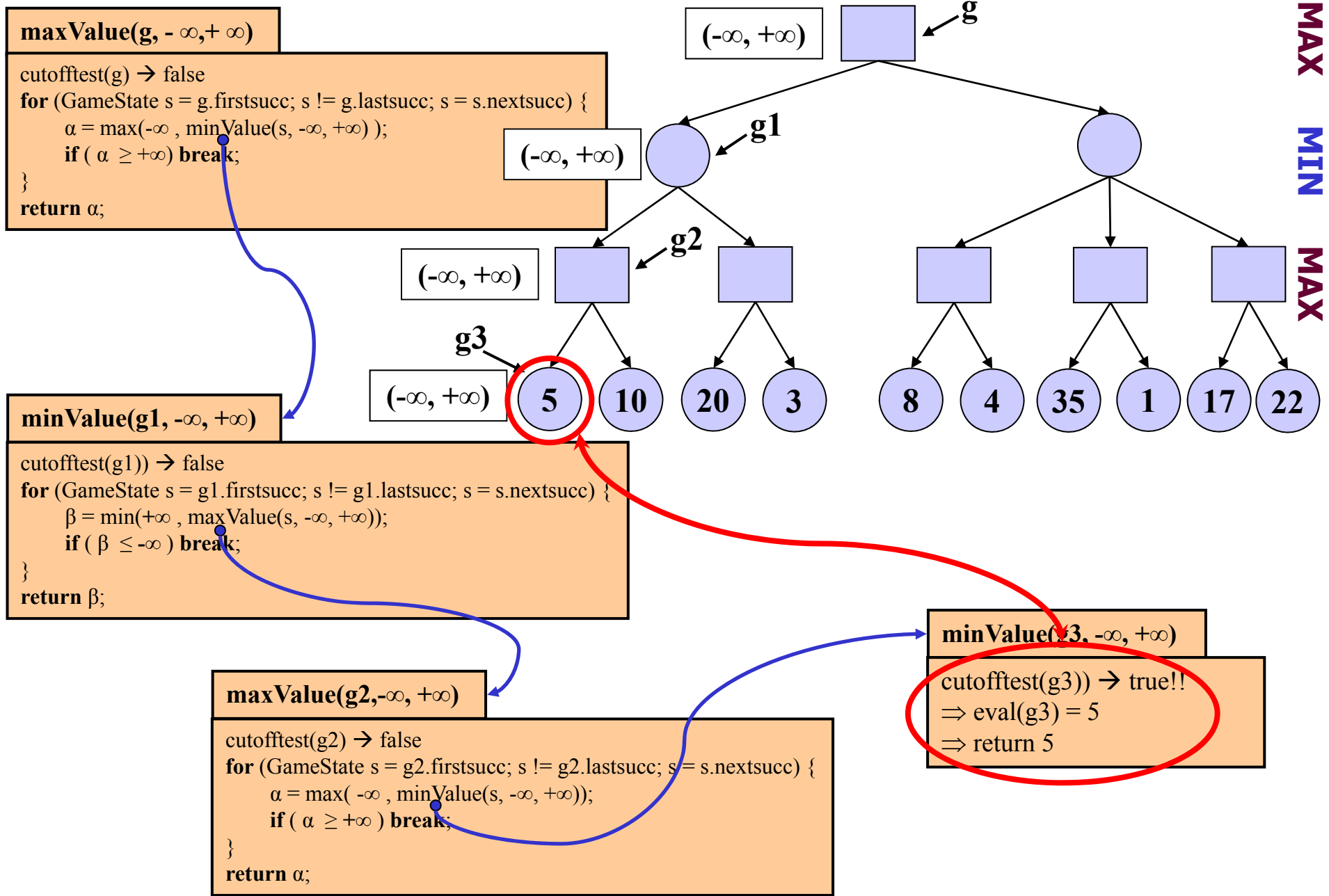
```
cutofftest(g1) → false  
for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {  
    β = min(+∞, maxValue(s, -∞, +∞));  
    if (β ≤ -∞) break;  
}  
return β;
```



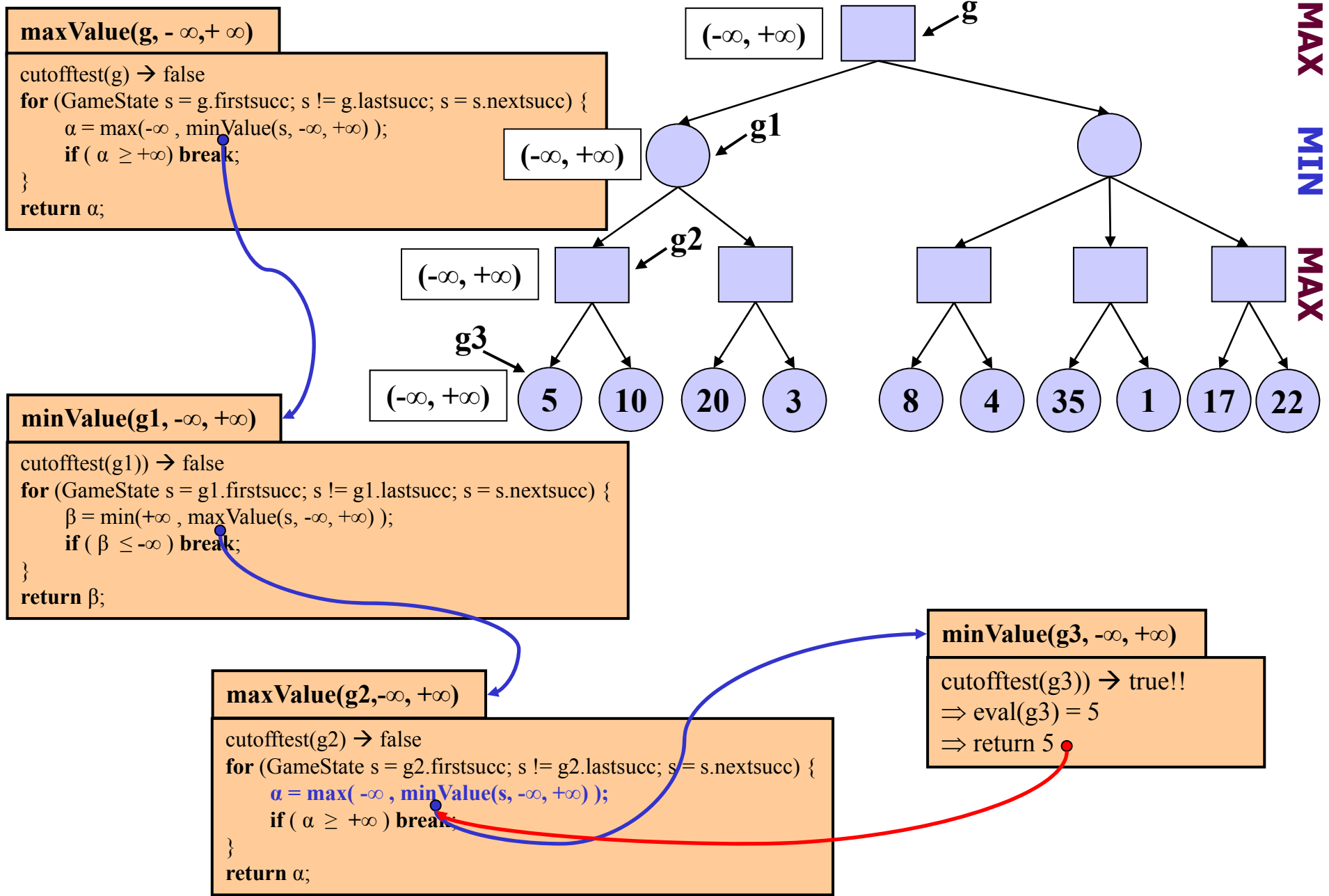


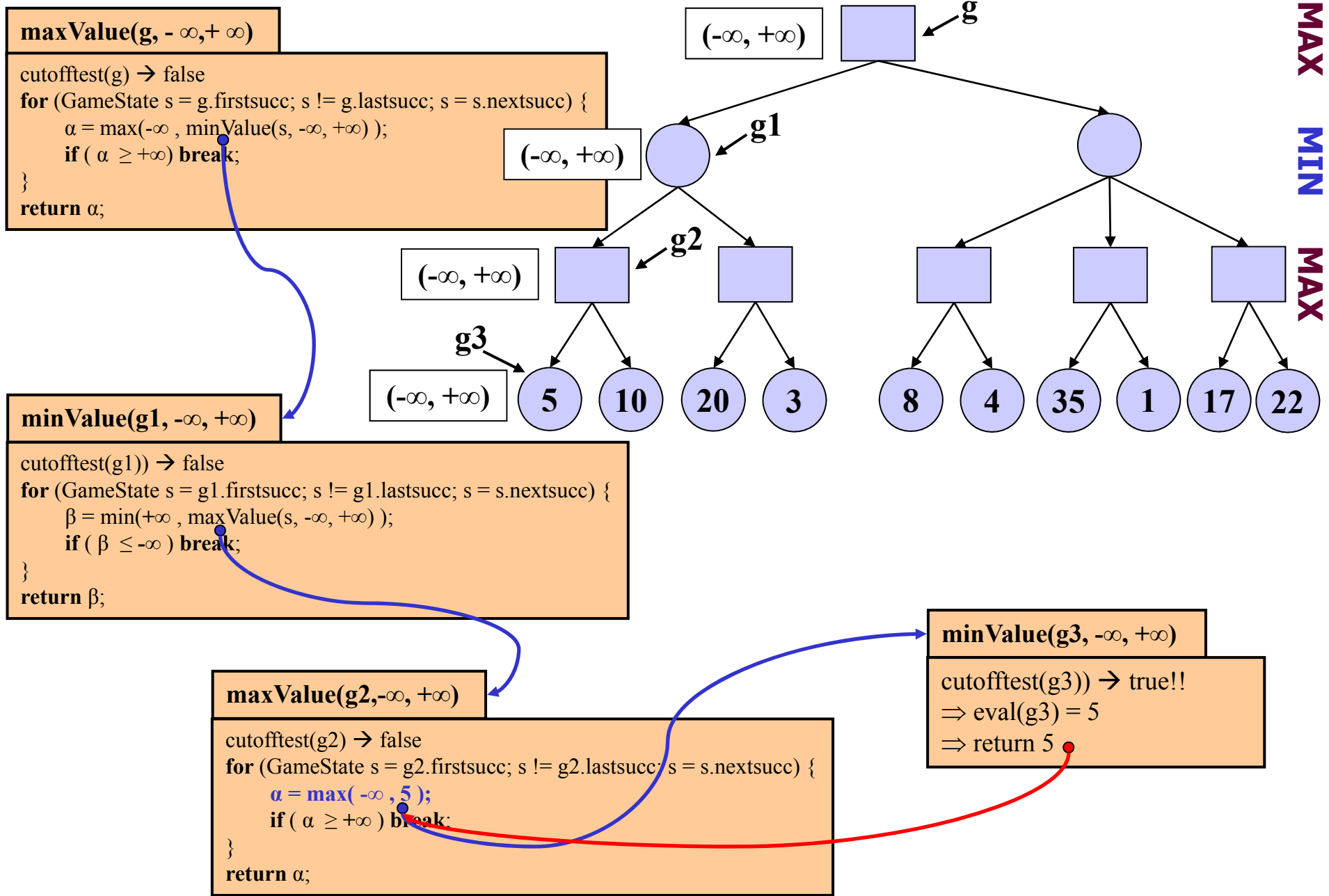


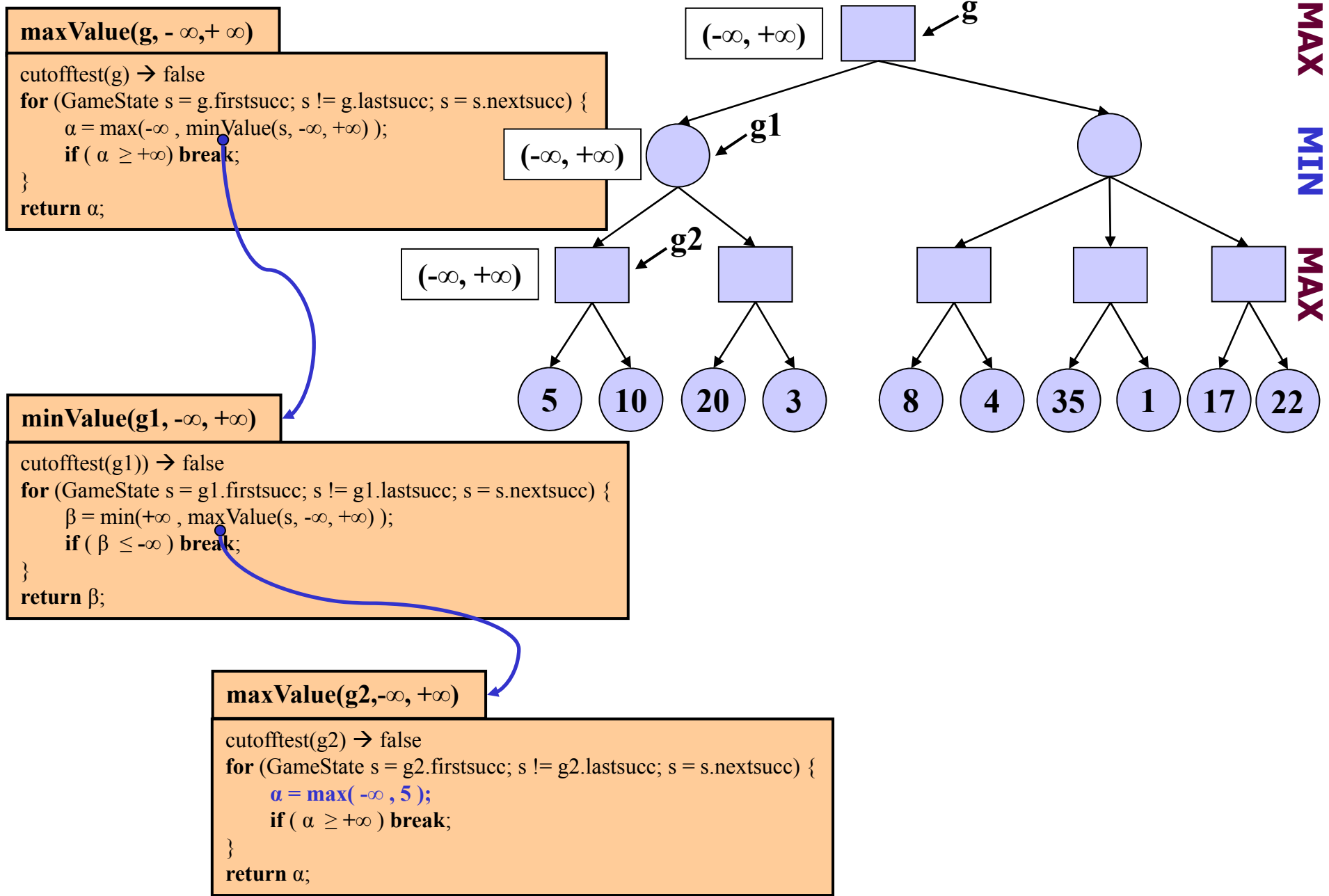


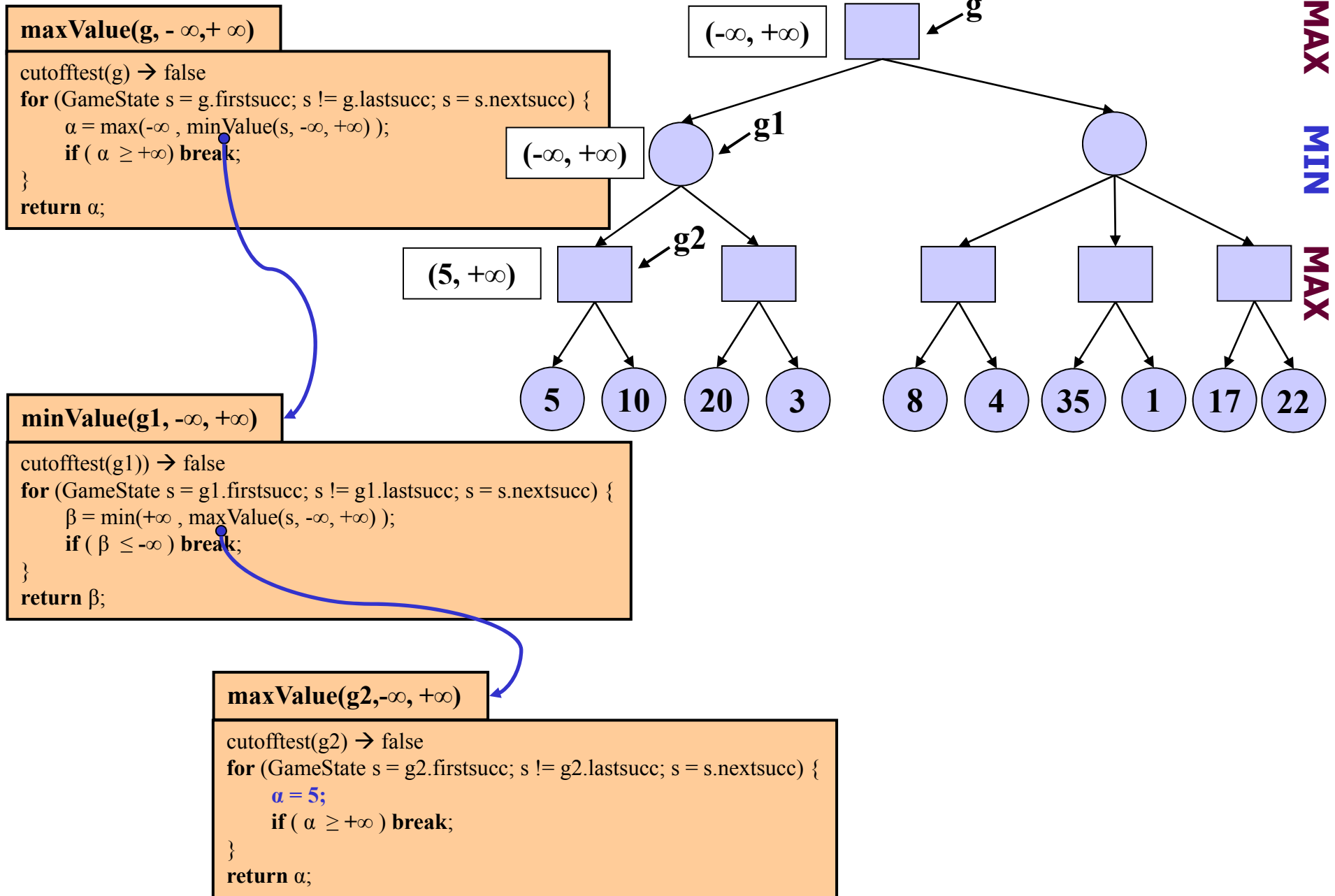


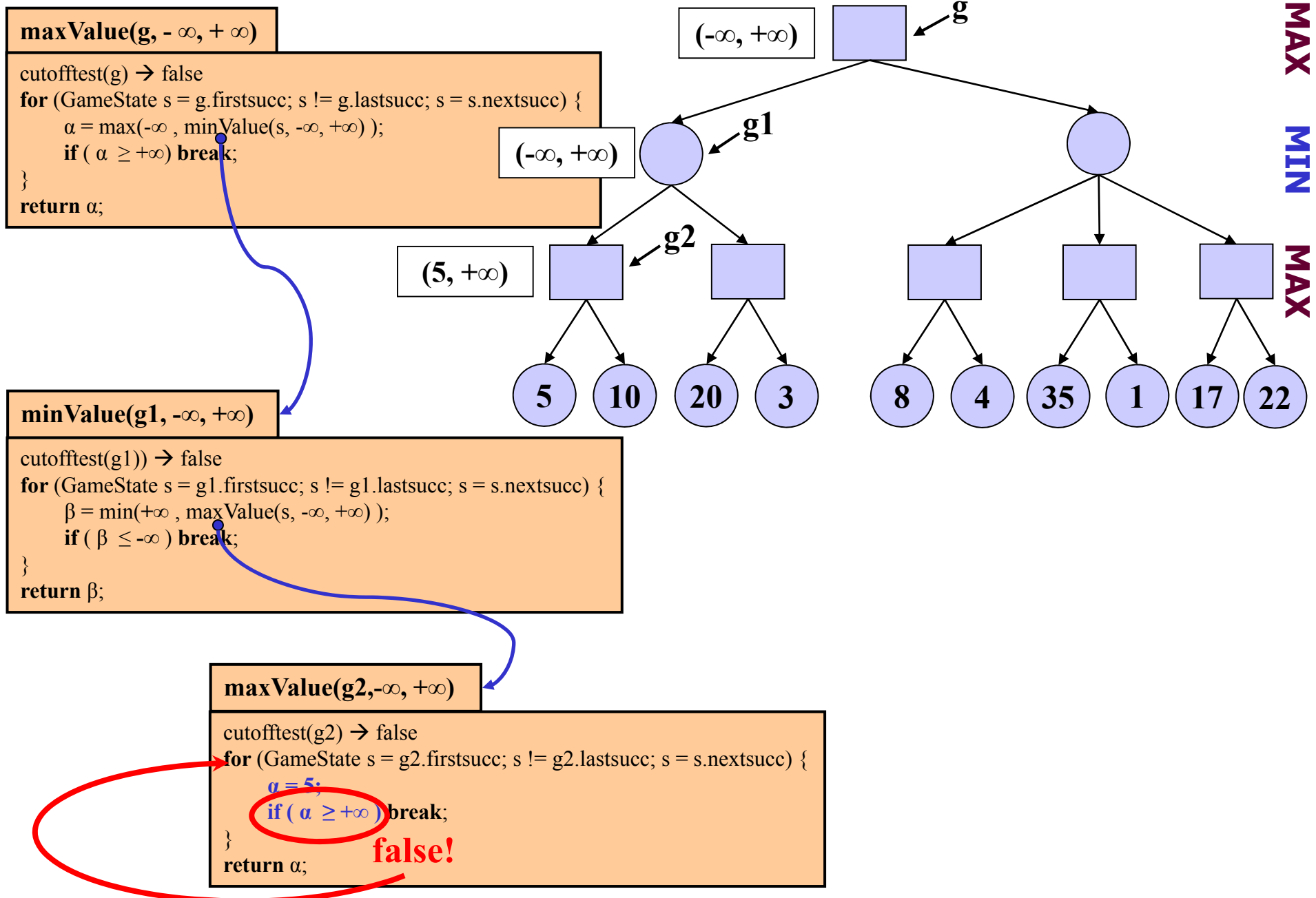


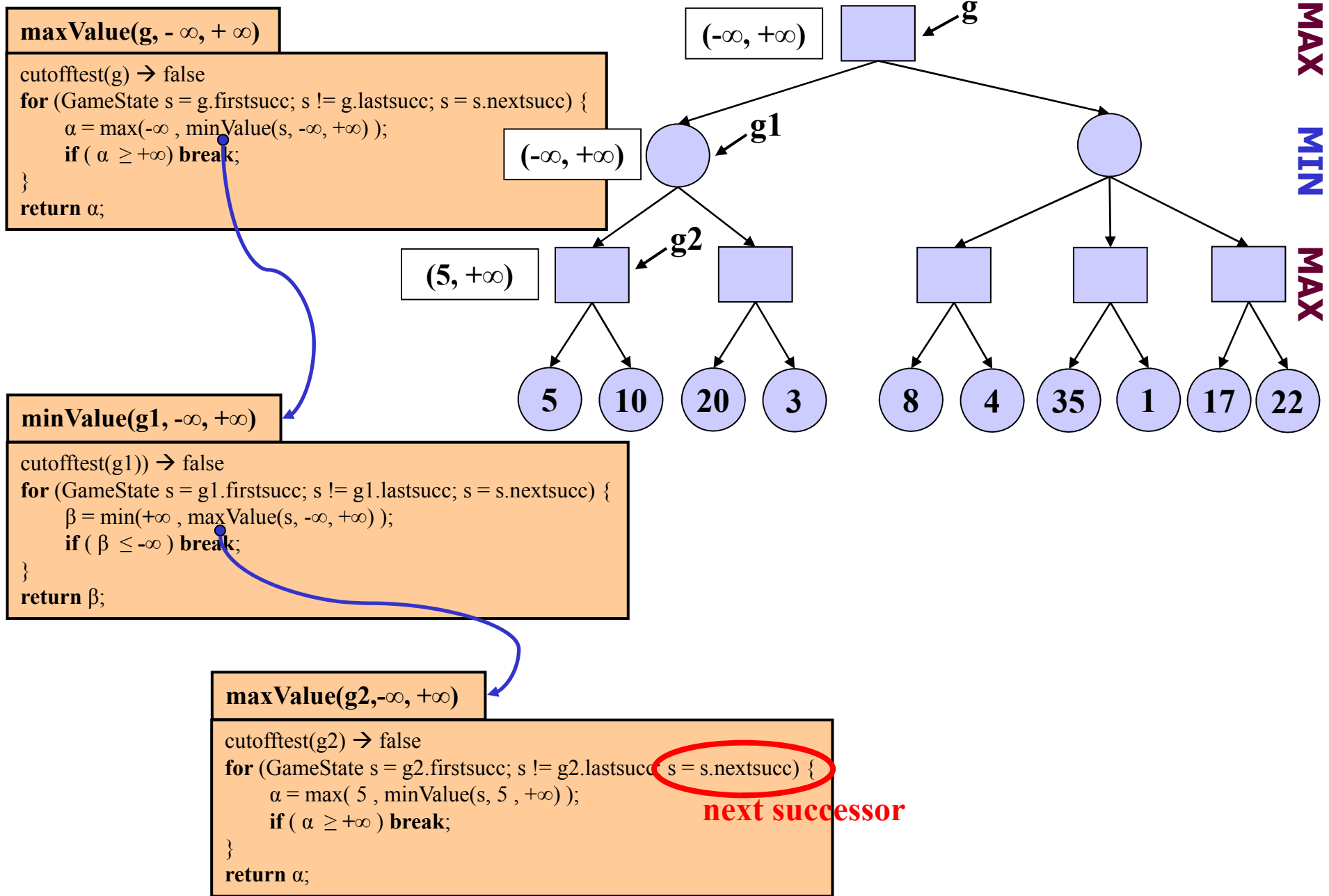


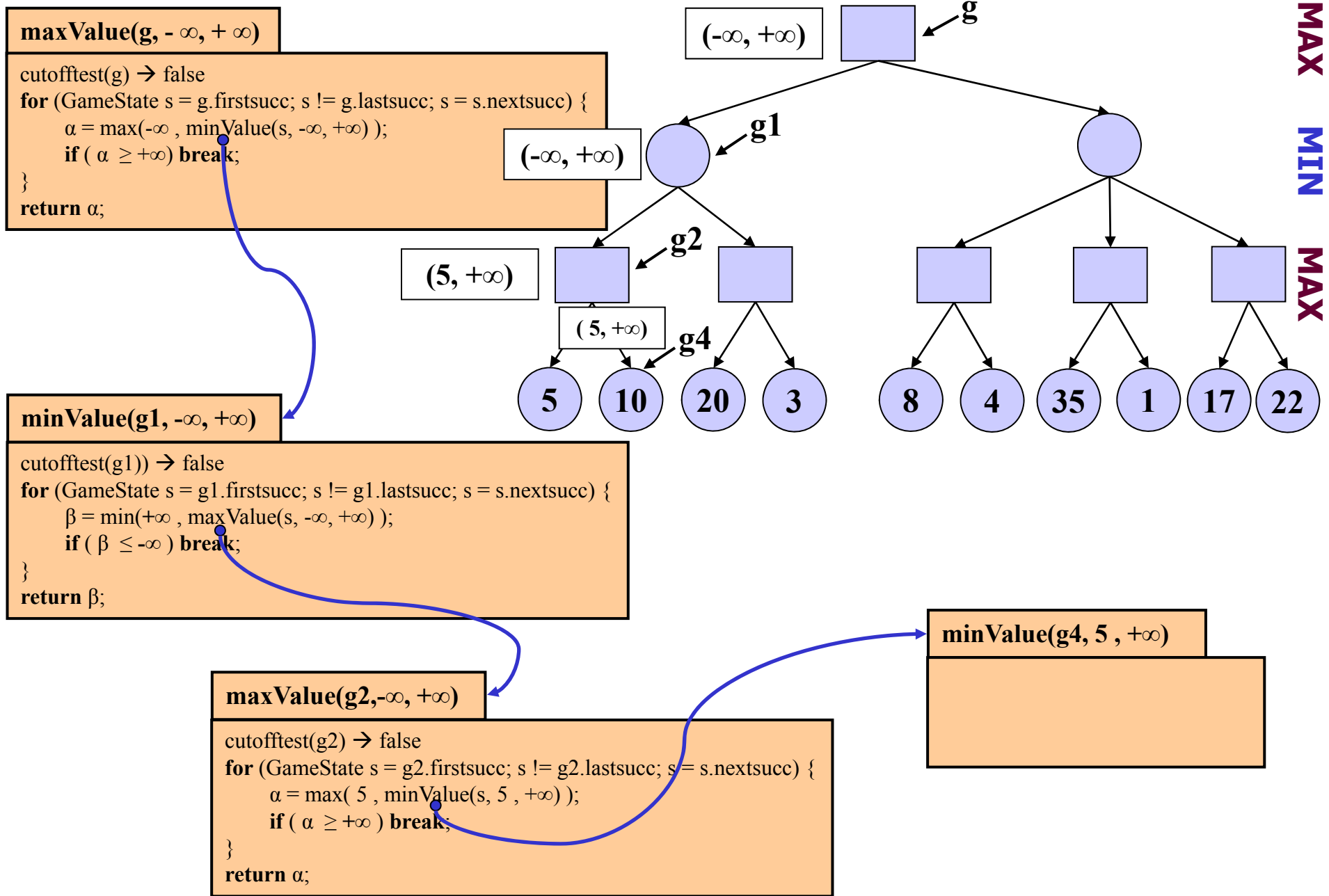


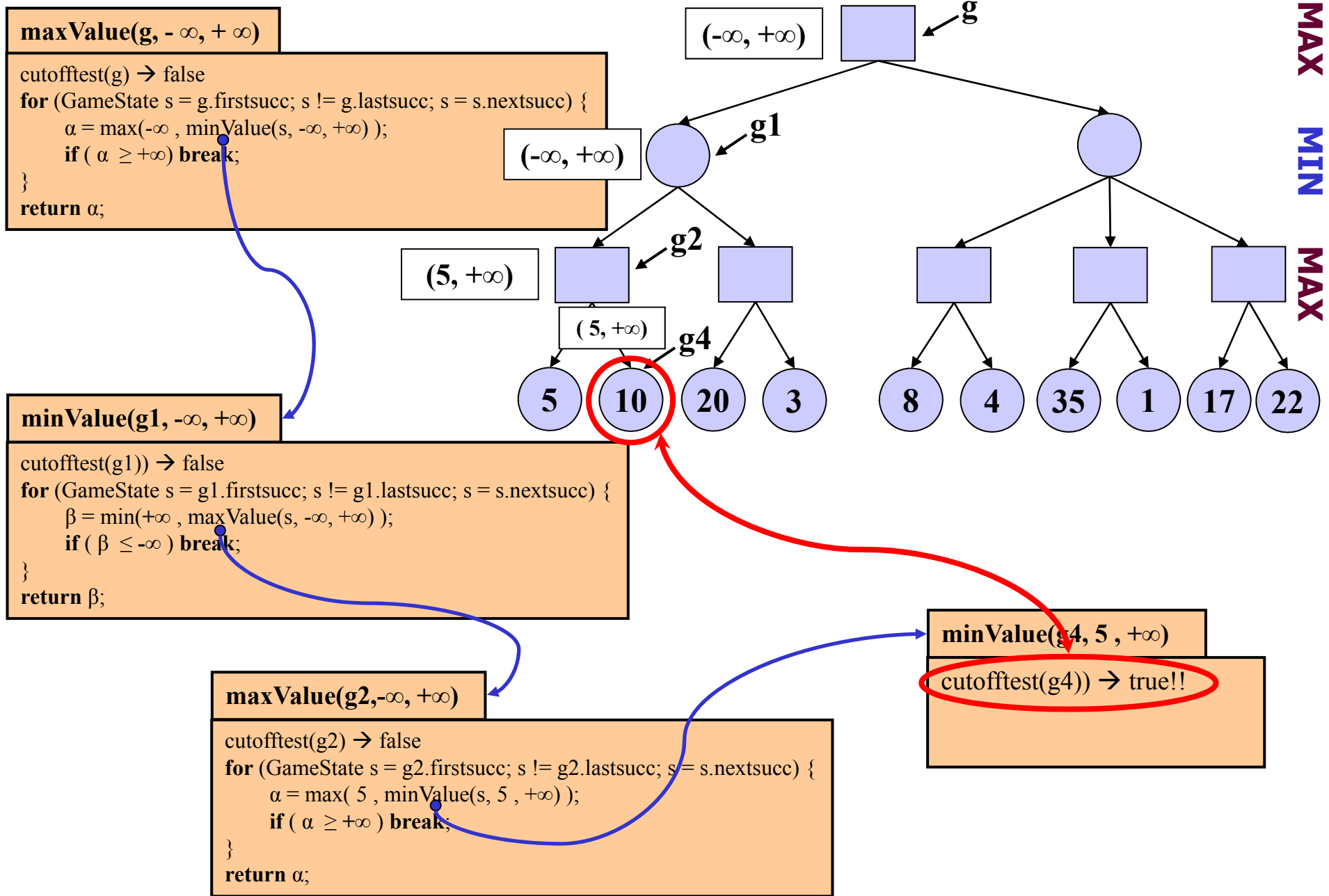




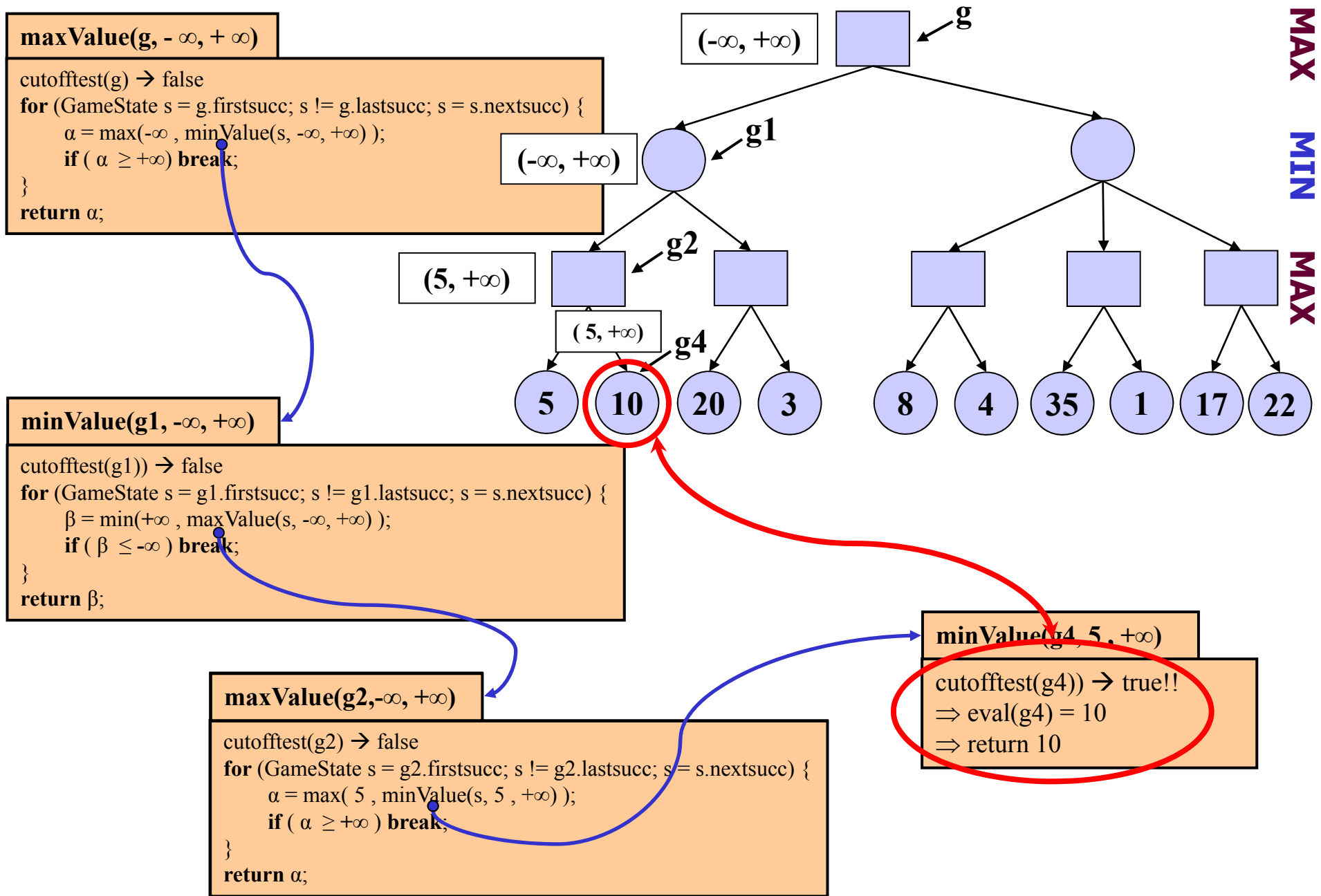


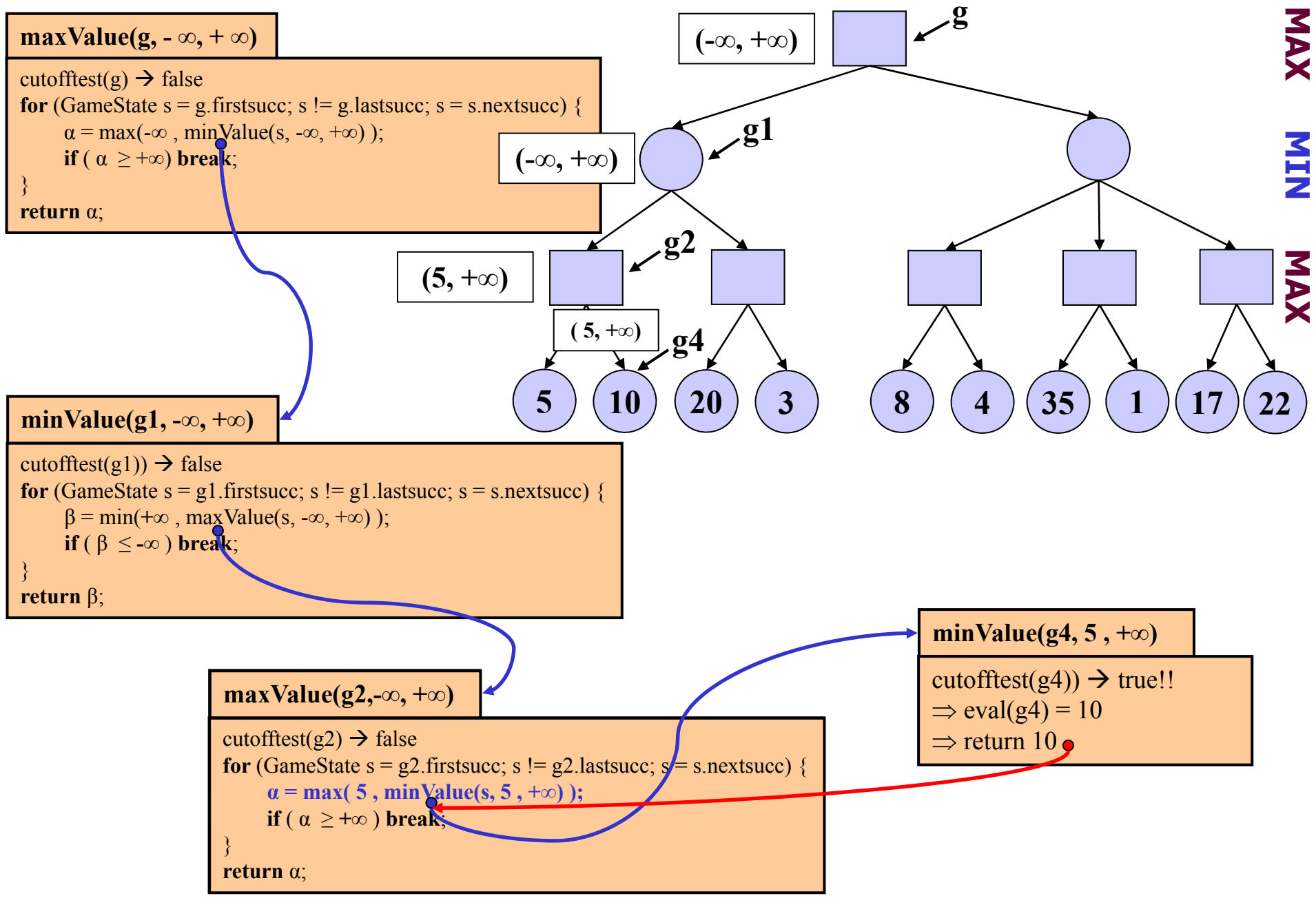


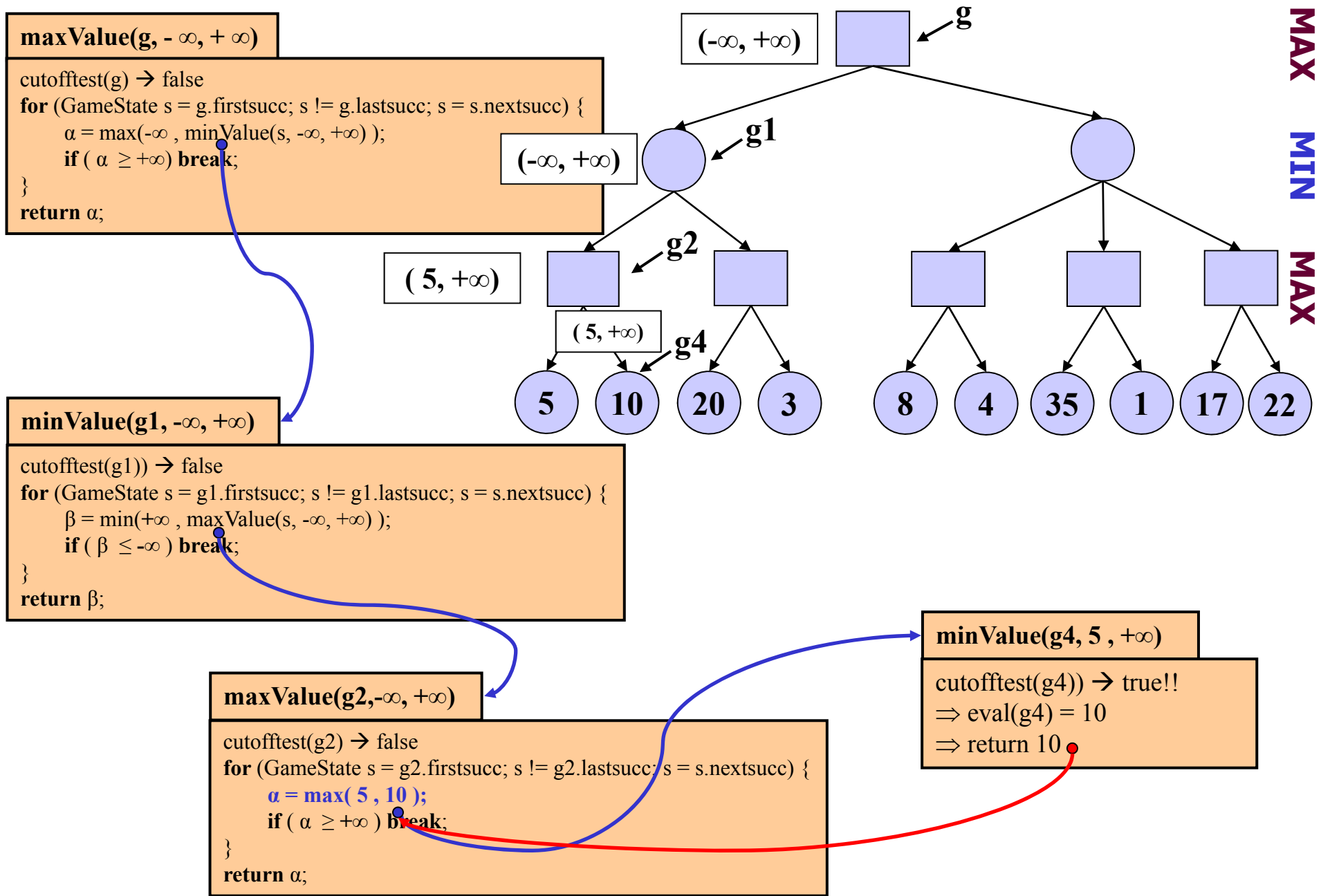


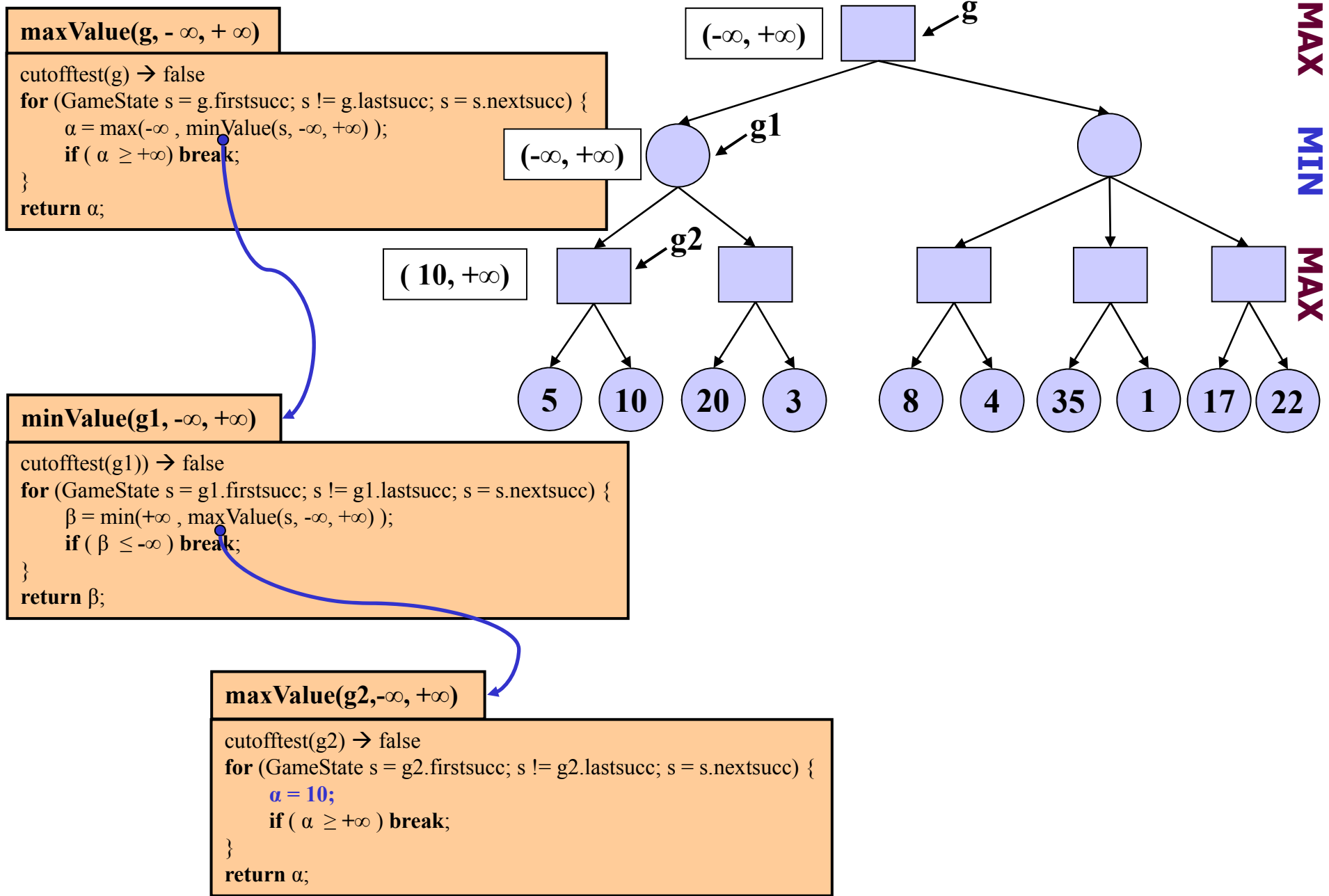


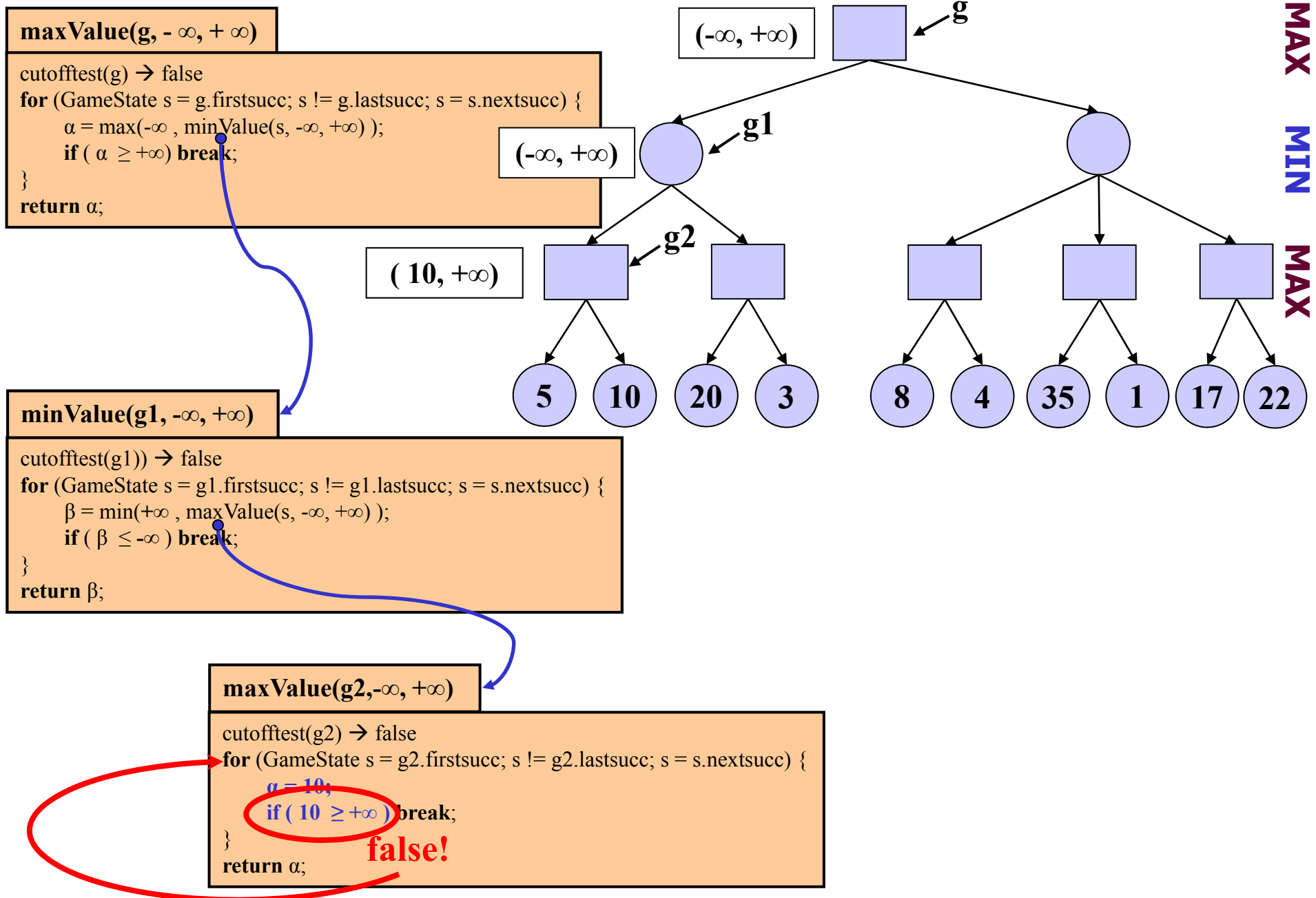


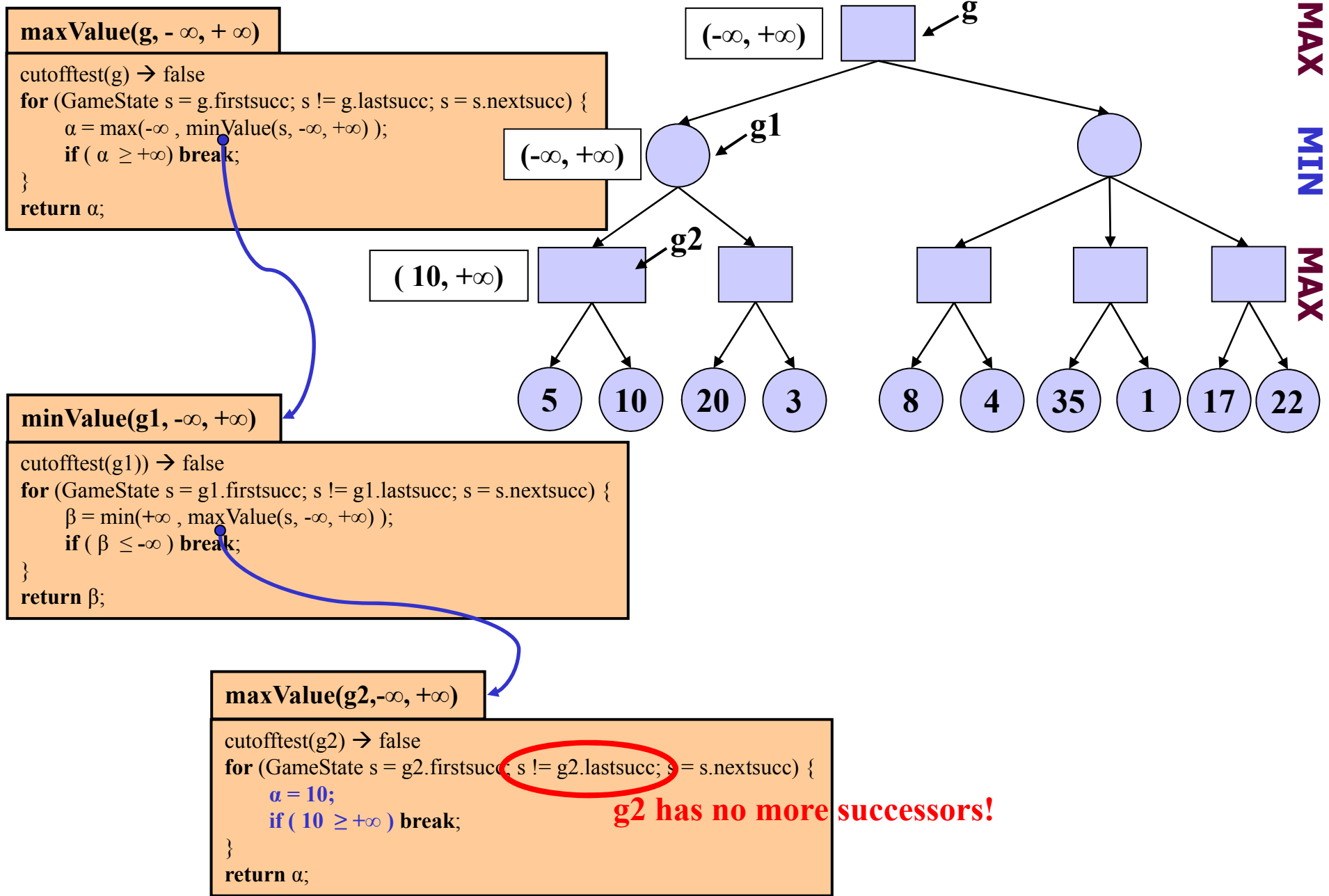


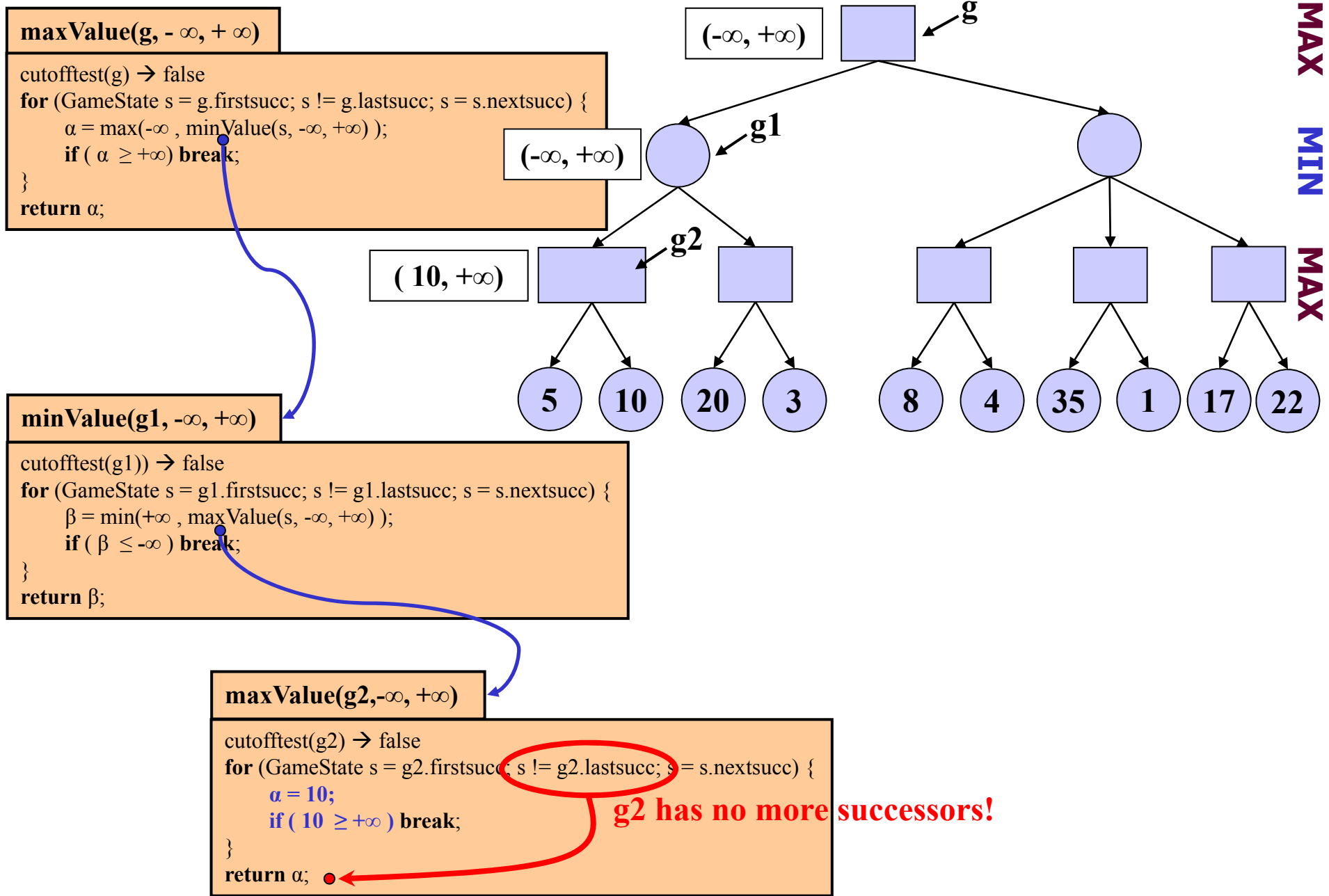


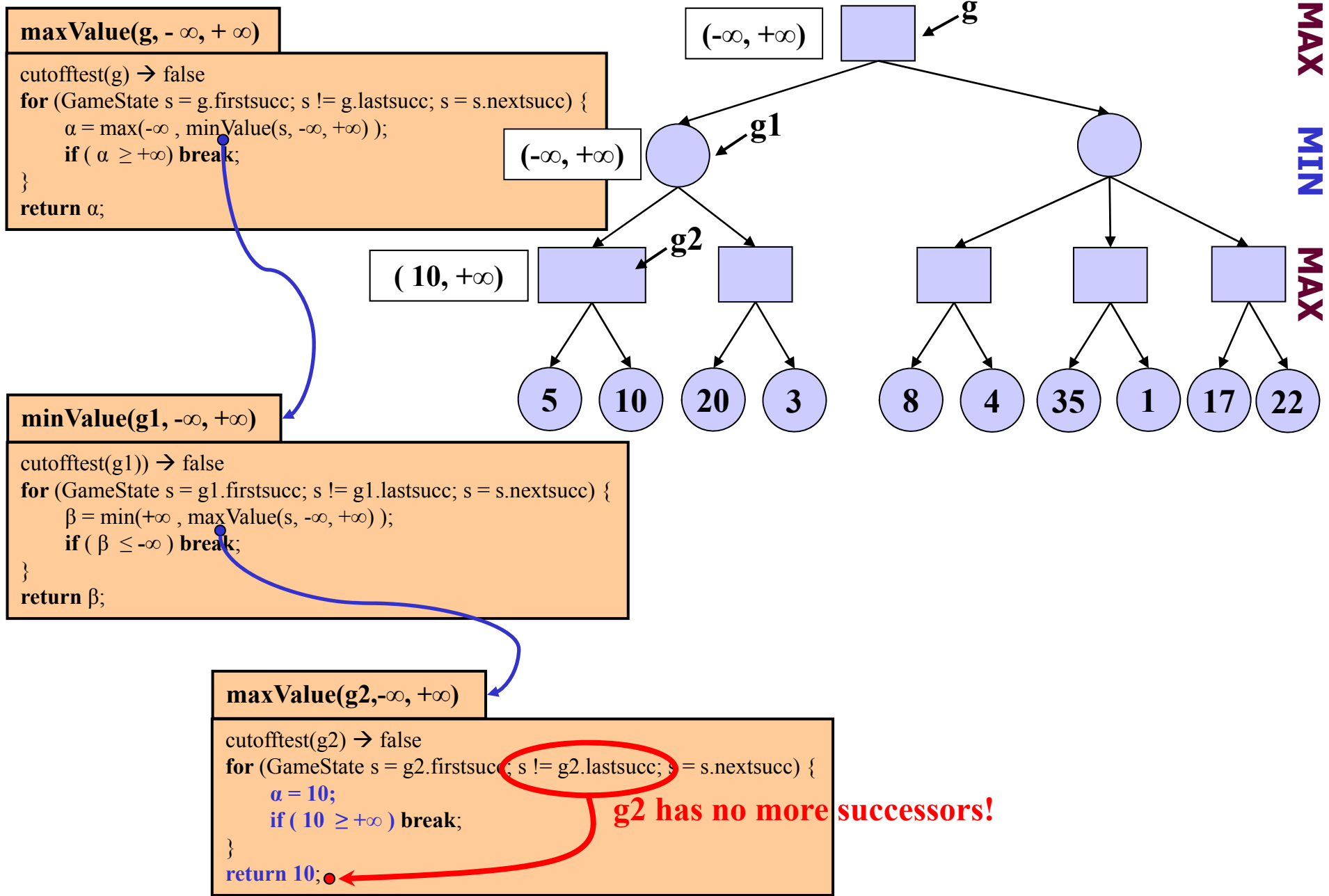














```
maxValue(g, -∞, +∞)
```

```
cutofftest(g) → false  
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {  
    α = max(-∞, minValue(s, -∞, +∞));  
    if (α ≥ +∞) break;  
}  
return α;
```

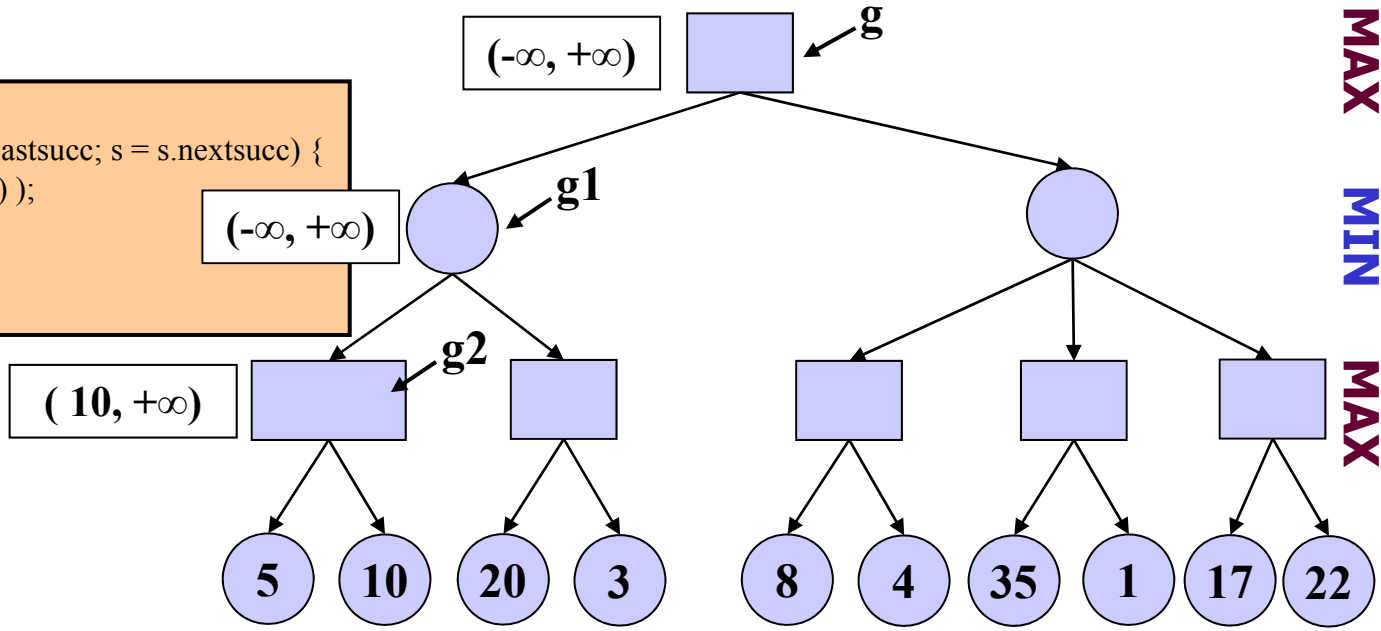
```
minValue(g1, -∞, +∞)
```

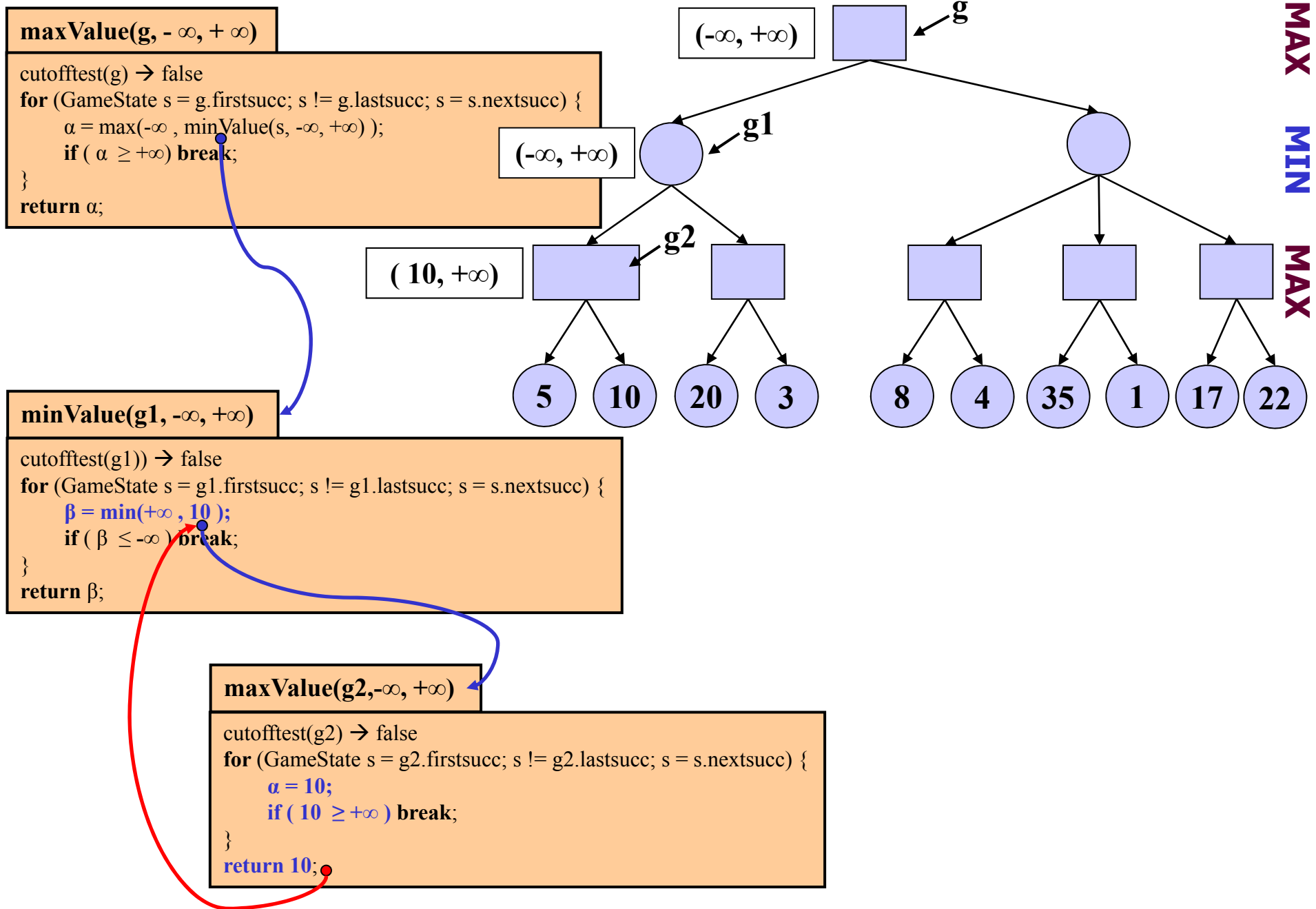
```
cutofftest(g1) → false  
for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {  
    β = min(+∞, maxValue(s, -∞, +∞));  
    if (β ≤ -∞) break;  
}  
return β;
```

```
maxValue(g2, -∞, +∞)
```

```
cutofftest(g2) → false  
for (GameState s = g2.firstsucc; s != g2.lastsucc; s = s.nextsucc) {  
    α = 10;  
    if (10 ≥ +∞) break;  
}  
return 10;
```

**g2 has no more successors!**



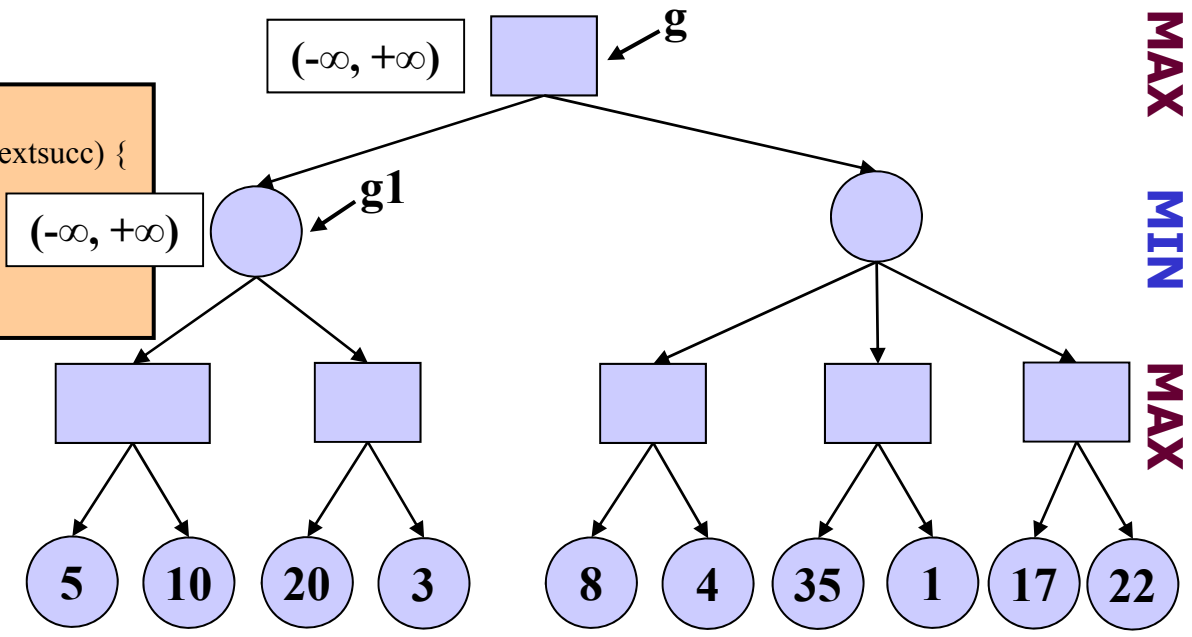


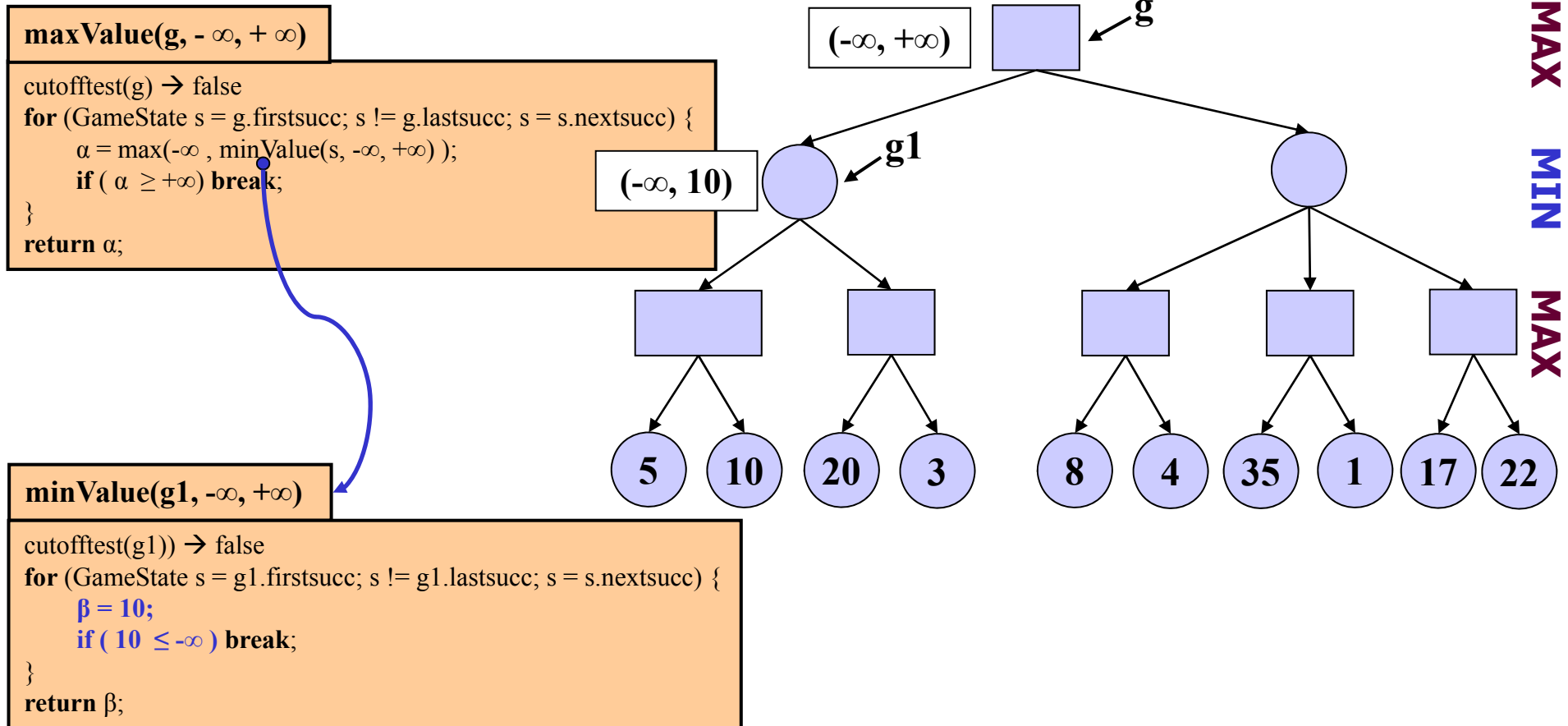
```
maxValue(g, -∞, +∞)
```

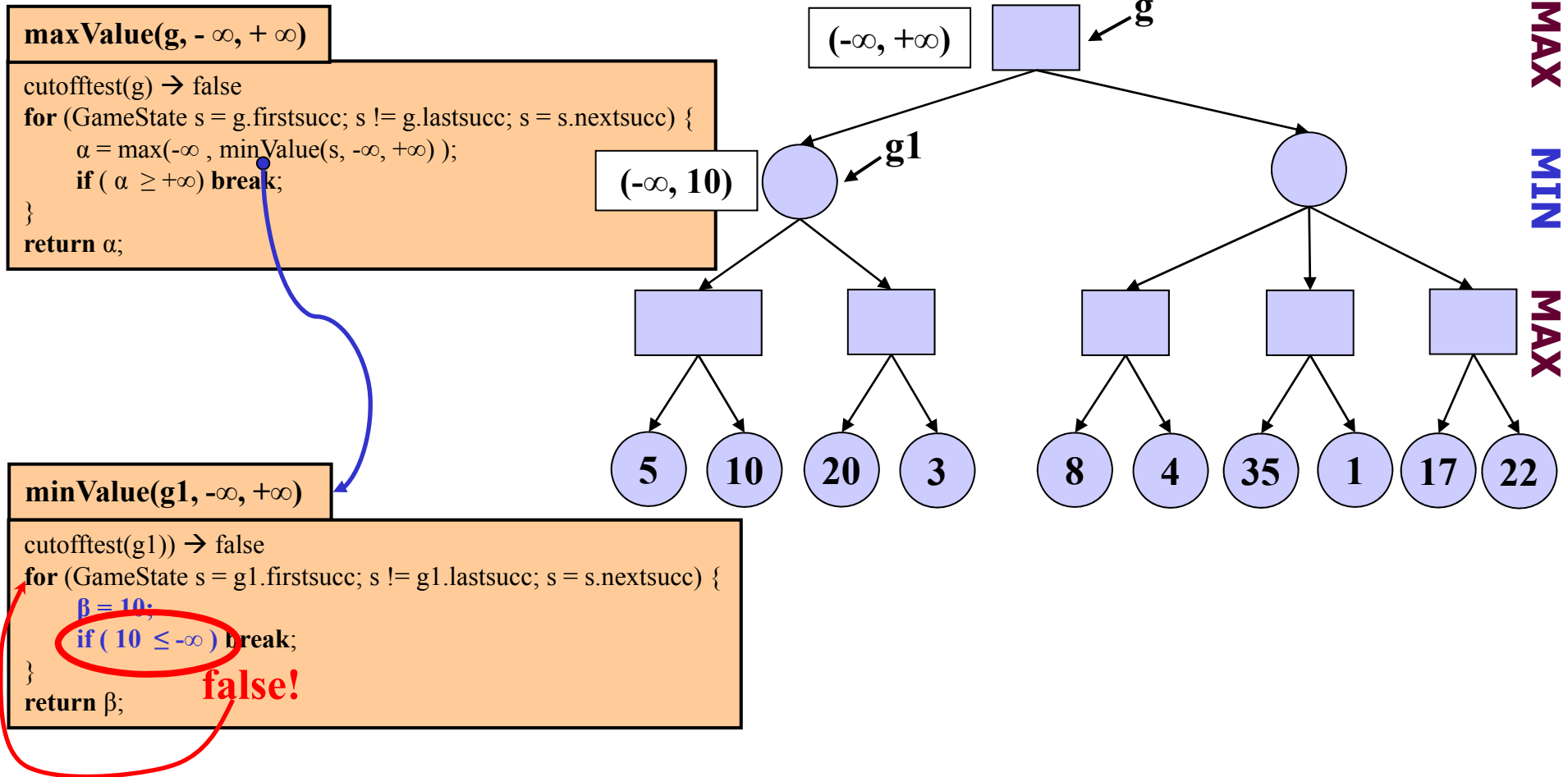
```
  cutofftest(g) → false  
  for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {  
    α = max(-∞, minValue(s, -∞, +∞));  
    if (α ≥ +∞) break;  
  }  
  return α;
```

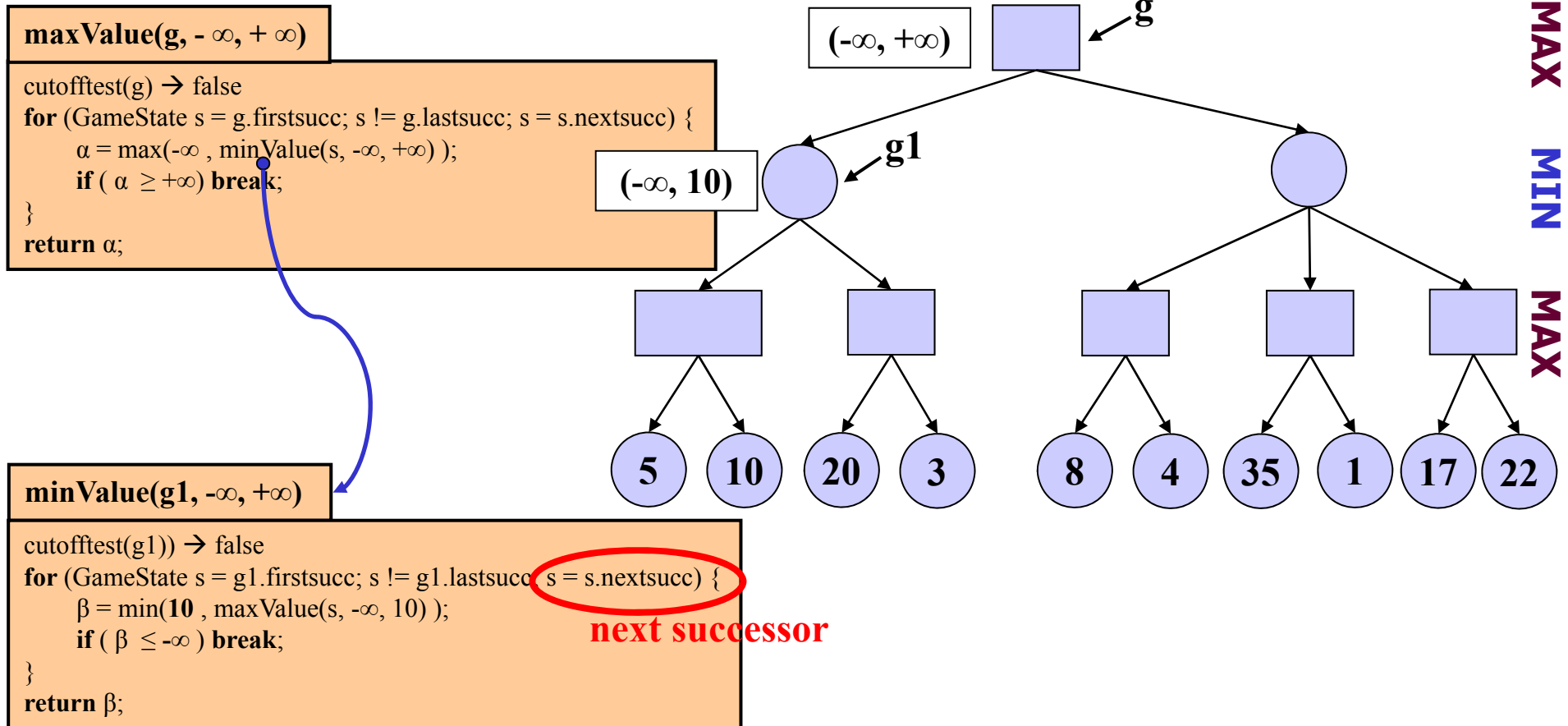
```
minValue(g1, -∞, +∞)
```

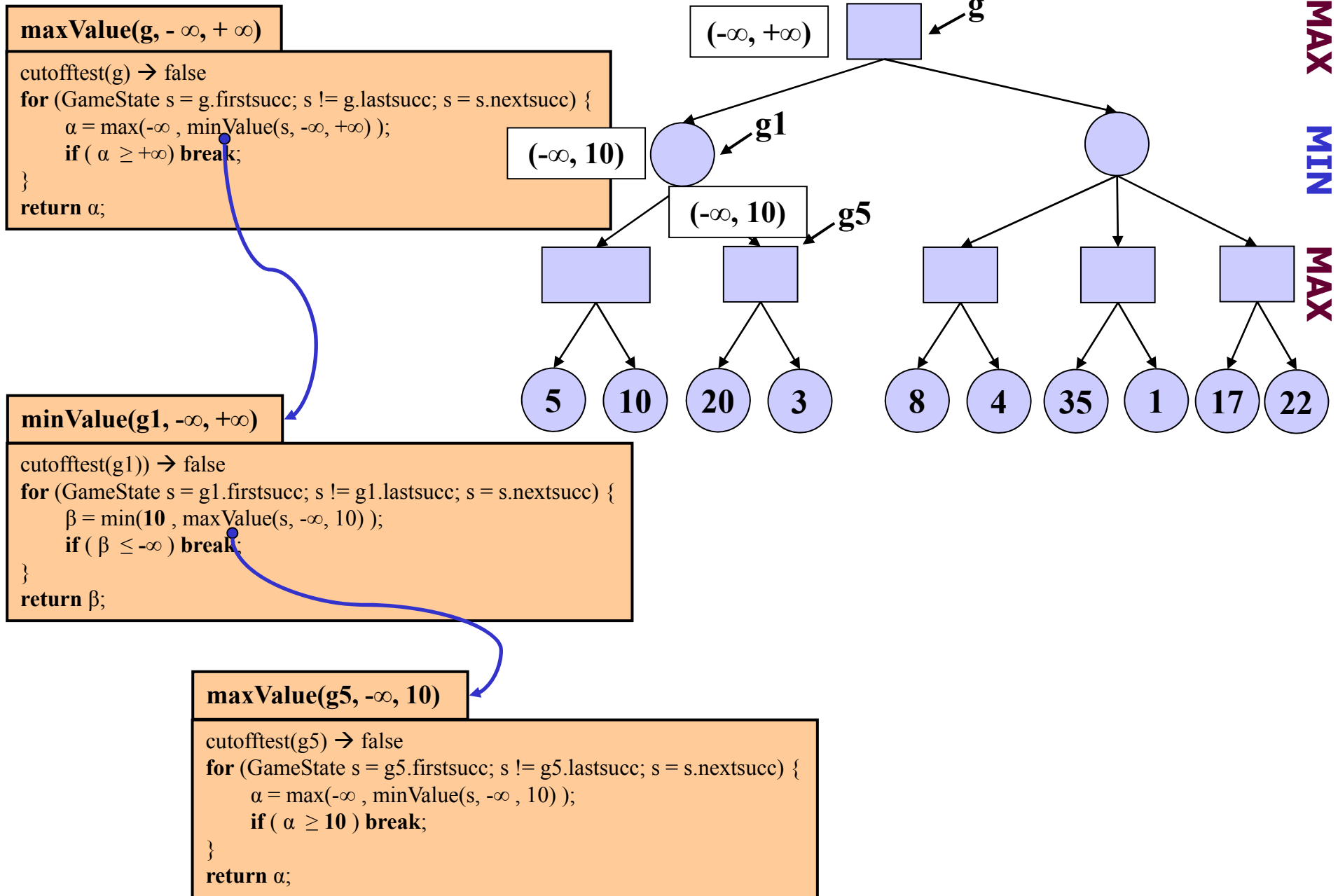
```
  cutofftest(g1) → false  
  for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {  
    β = min(+∞, 10);  
    if (β ≤ -∞) break;  
  }  
  return β;
```











```
maxValue(g, -∞, +∞)
```

```
  cutofftest(g) → false  
  for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {  
    α = max(-∞, minValue(s, -∞, +∞));  
    if (α ≥ +∞) break;  
  }  
  return α;
```

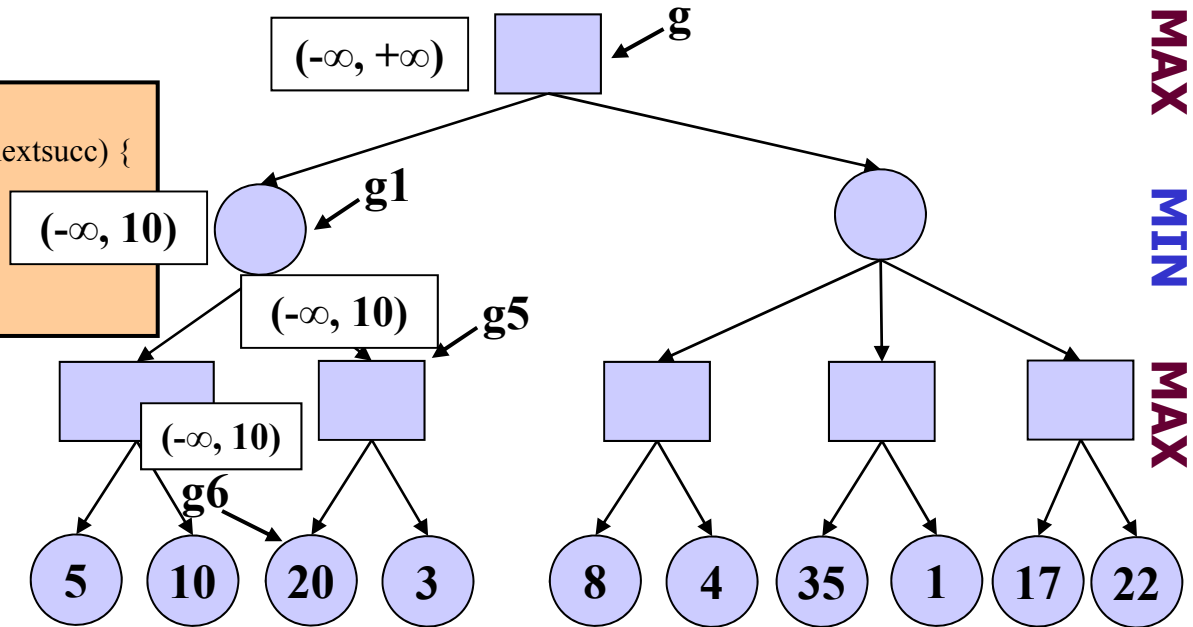
```
minValue(g1, -∞, +∞)
```

```
  cutofftest(g1) → false  
  for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {  
    β = min(10, maxValue(s, -∞, 10));  
    if (β ≤ -∞) break;  
  }  
  return β;
```

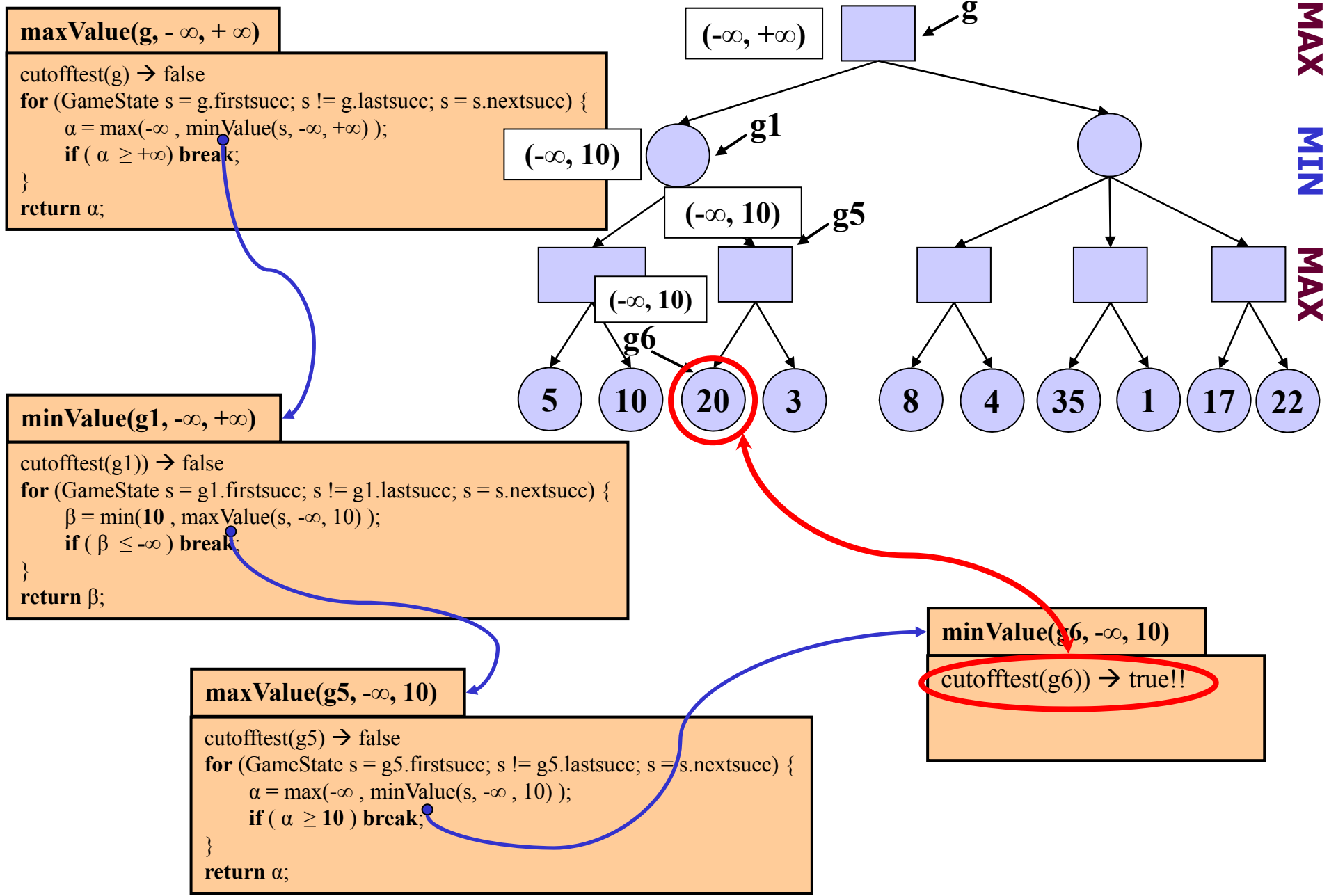
```
maxValue(g5, -∞, 10)
```

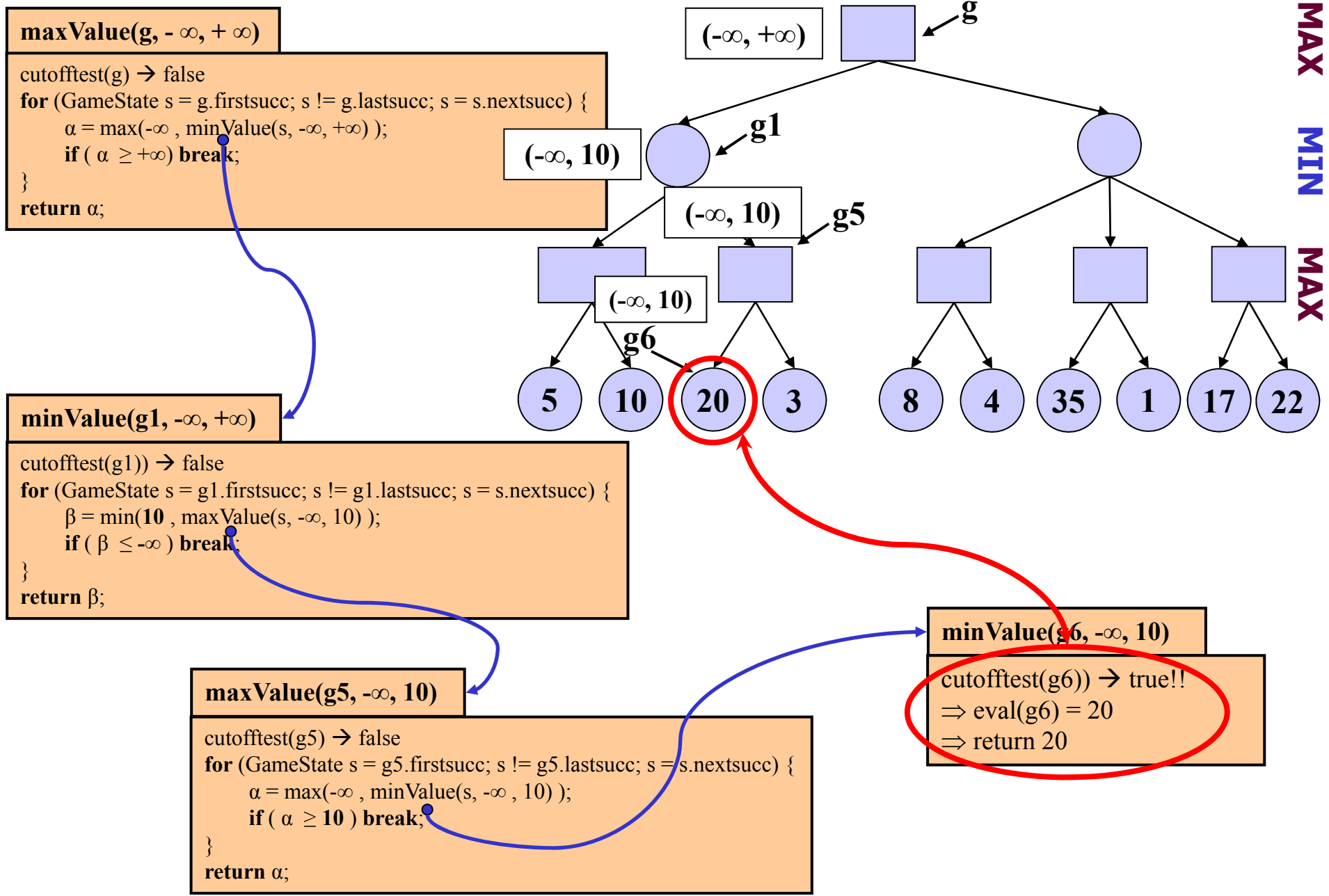
```
  cutofftest(g5) → false  
  for (GameState s = g5.firstsucc; s != g5.lastsucc; s = s.nextsucc) {  
    α = max(-∞, minValue(s, -∞, 10));  
    if (α ≥ 10) break;  
  }  
  return α;
```

```
minValue(g6, -∞, 10)
```









```
maxValue(g, -∞, +∞)
```

```
cutofftest(g) → false  
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {  
    α = max(-∞, minValue(s, -∞, +∞));  
    if (α ≥ +∞) break;  
}  
return α;
```

```
minValue(g1, -∞, +∞)
```

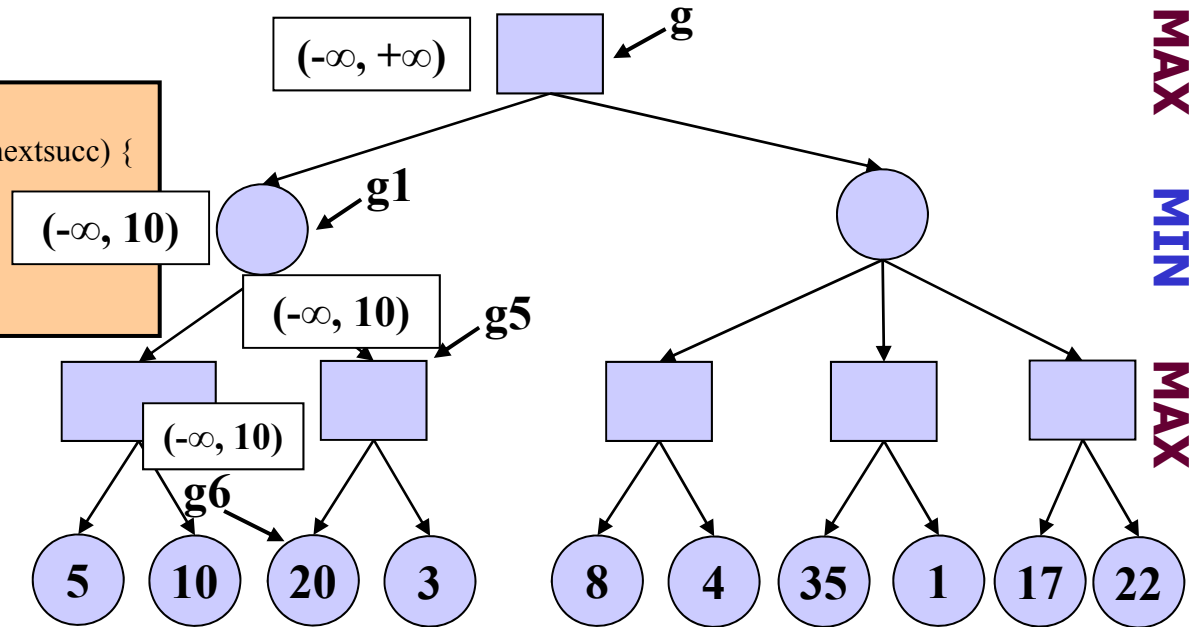
```
cutofftest(g1) → false  
for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {  
    β = min(10, maxValue(s, -∞, 10));  
    if (β ≤ -∞) break;  
}  
return β;
```

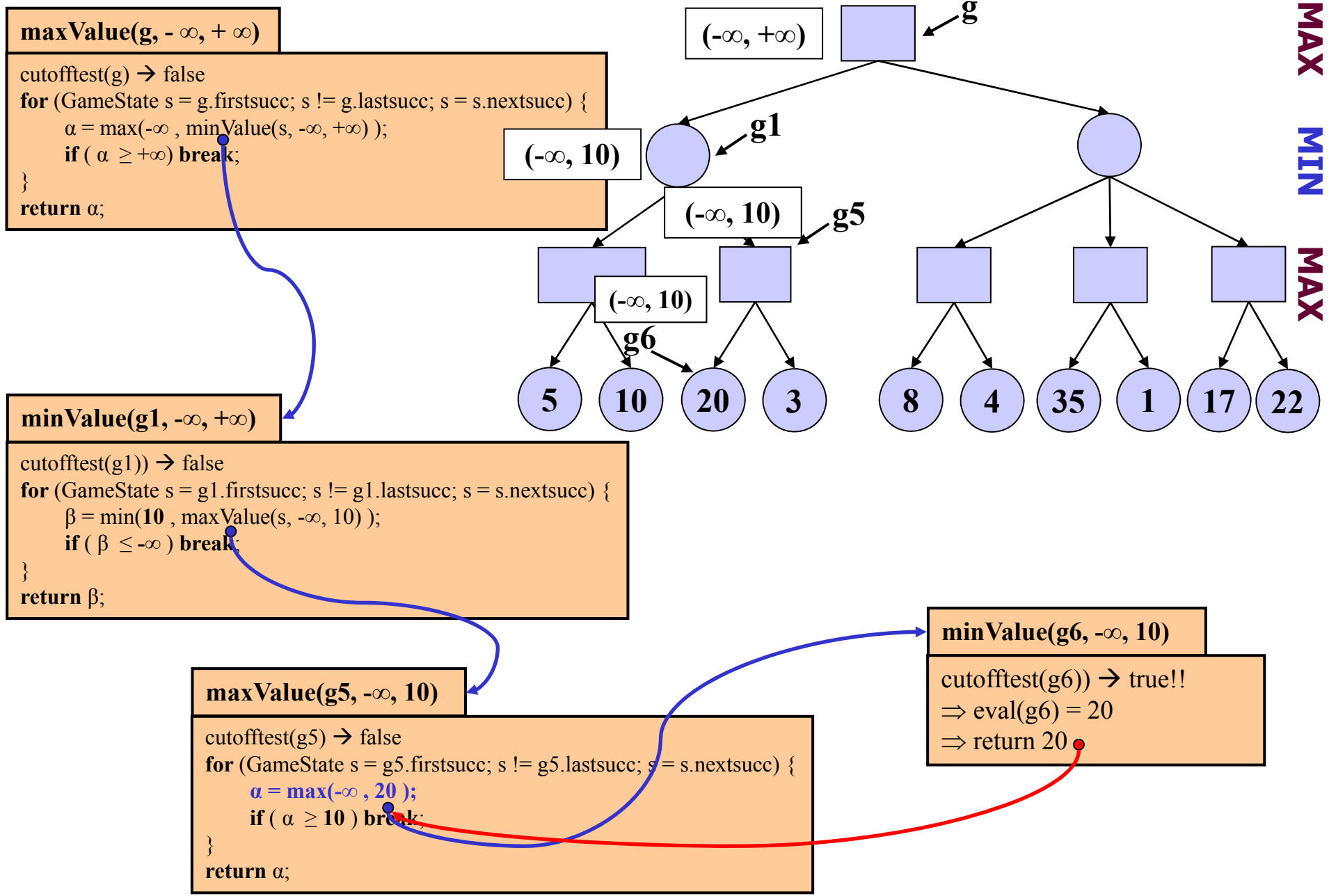
```
maxValue(g5, -∞, 10)
```

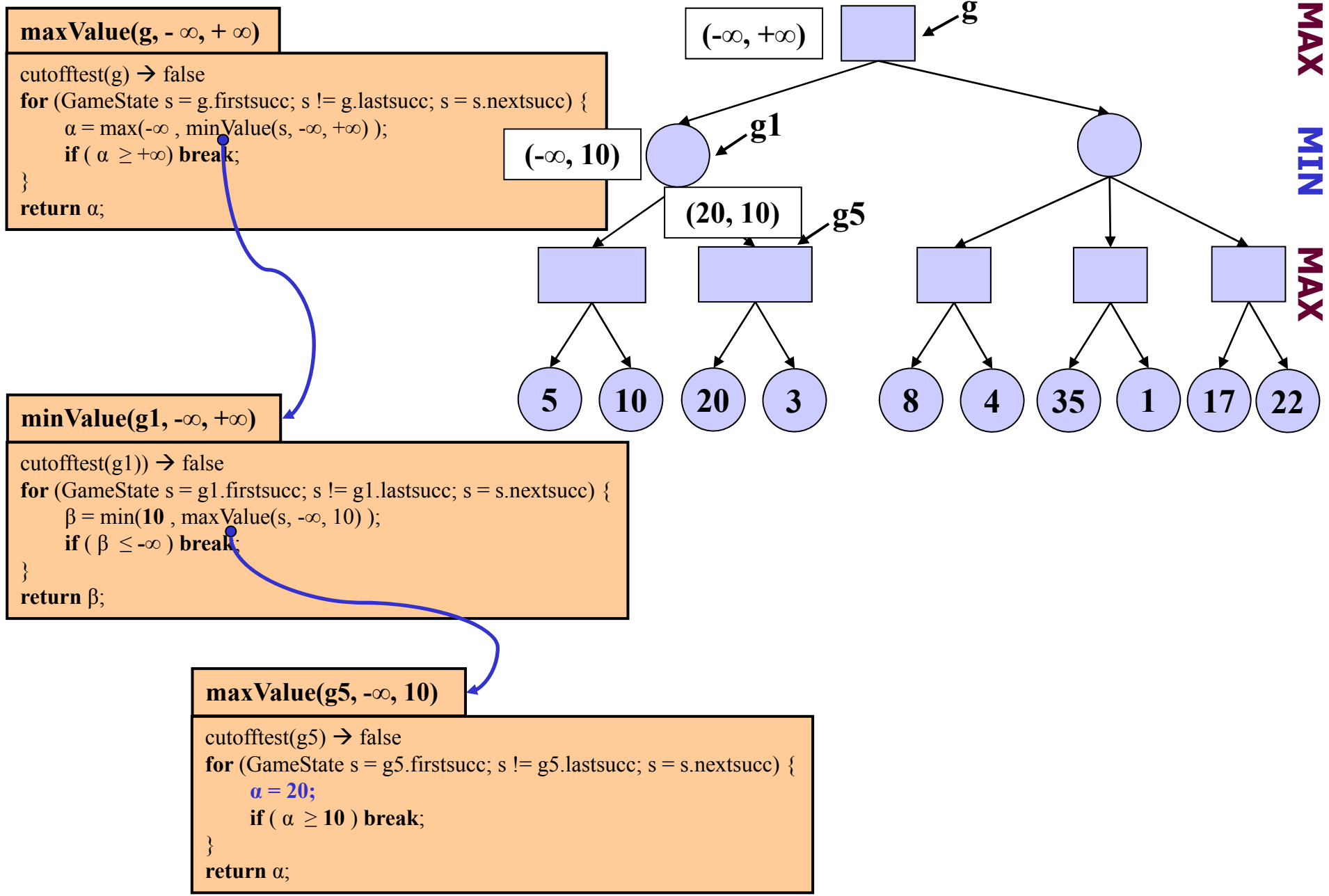
```
cutofftest(g5) → false  
for (GameState s = g5.firstsucc; s != g5.lastsucc; s = s.nextsucc) {  
    α = max(-∞, minValue(s, -∞, 10));  
    if (α ≥ 10) break;  
}  
return α;
```

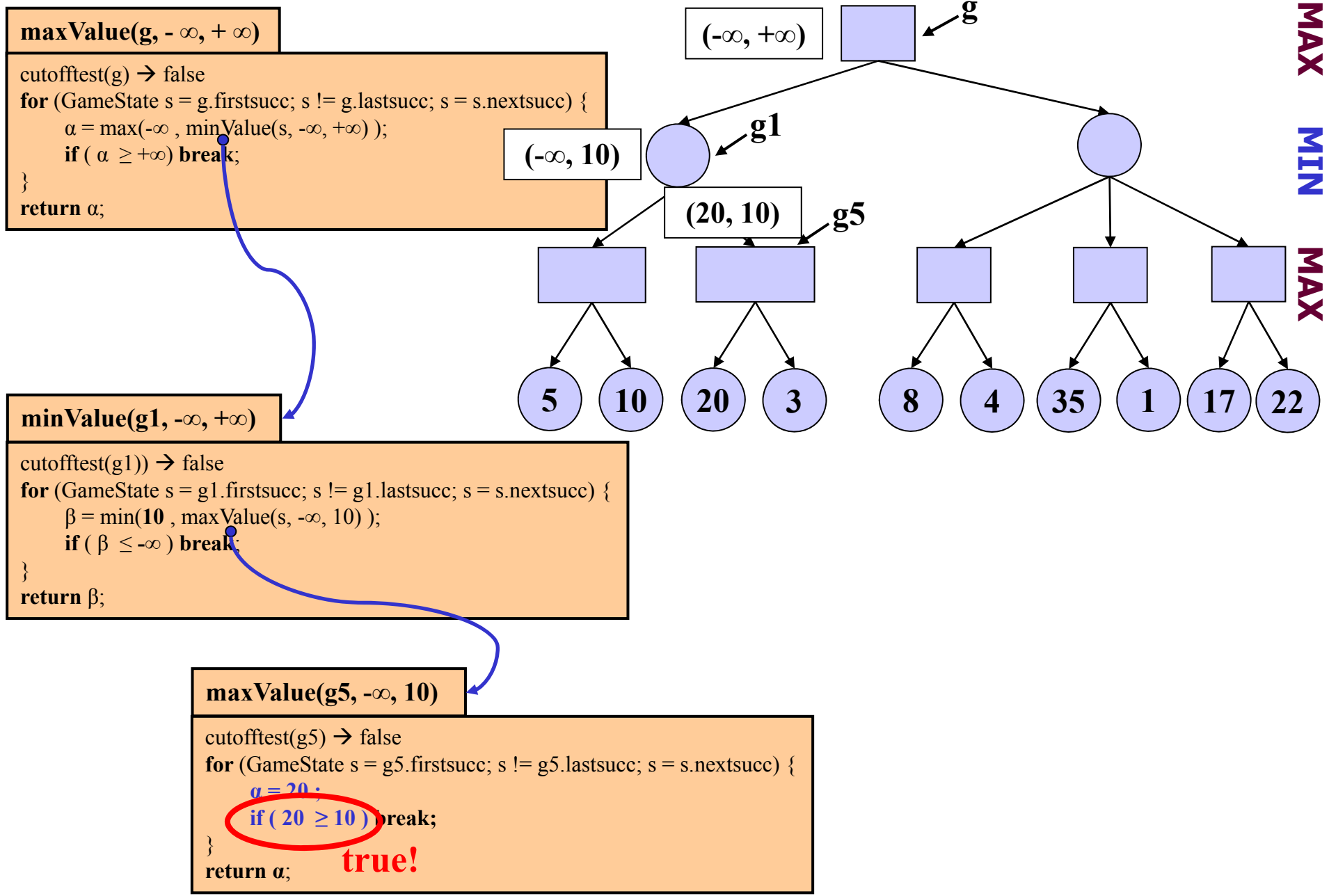
```
minValue(g6, -∞, 10)
```

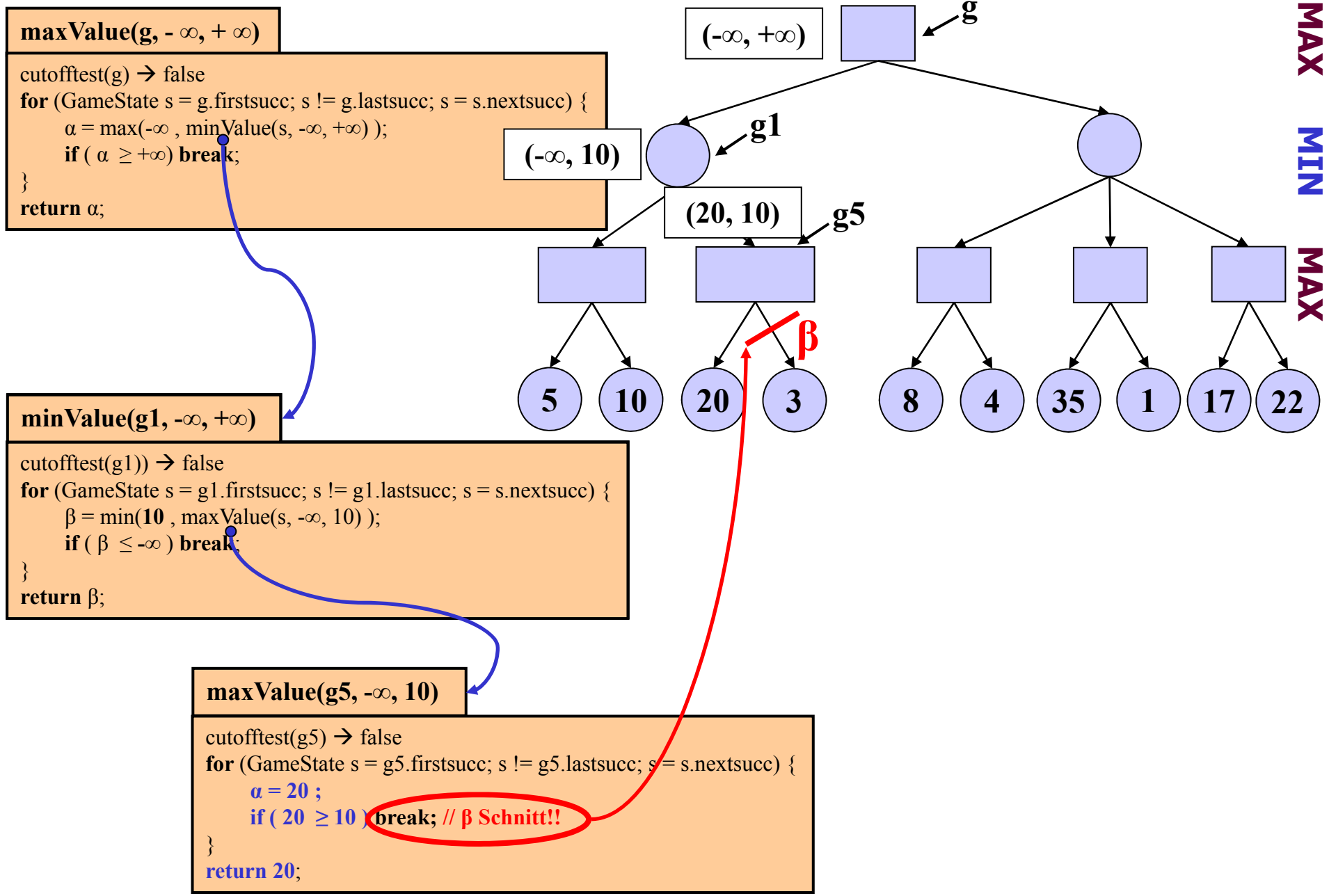
```
cutofftest(g6) → true!!  
⇒ eval(g6) = 20  
⇒ return 20
```

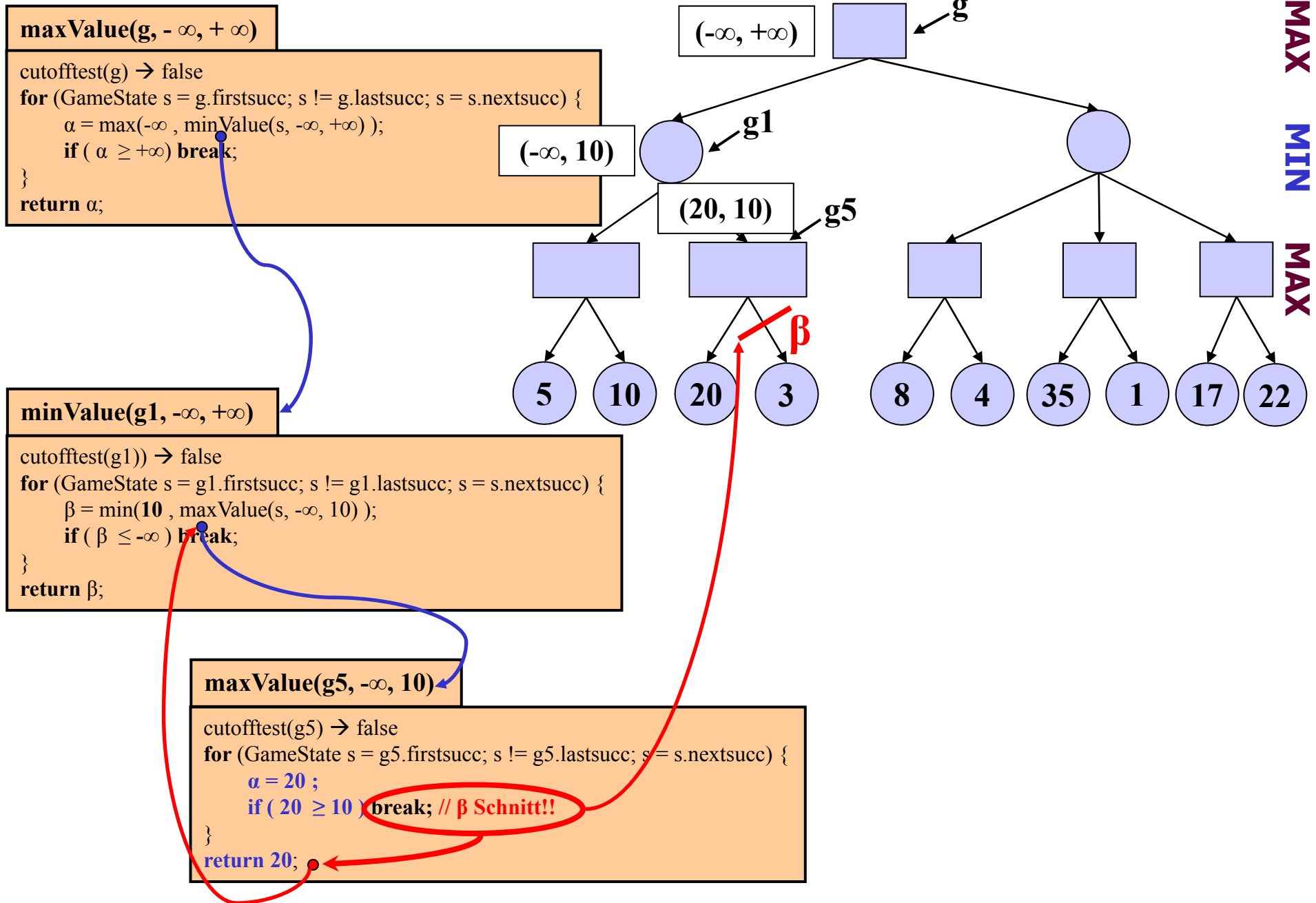




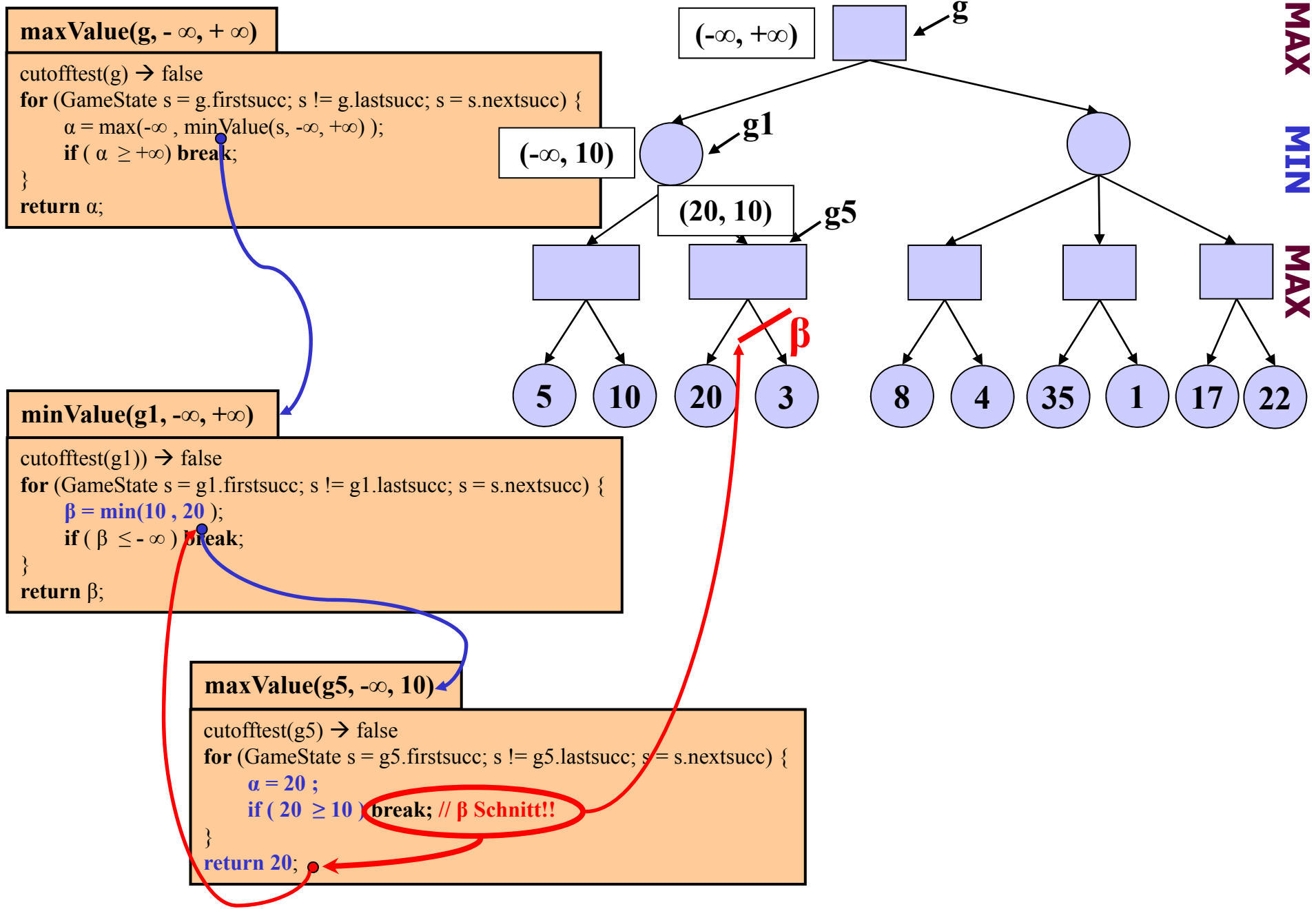


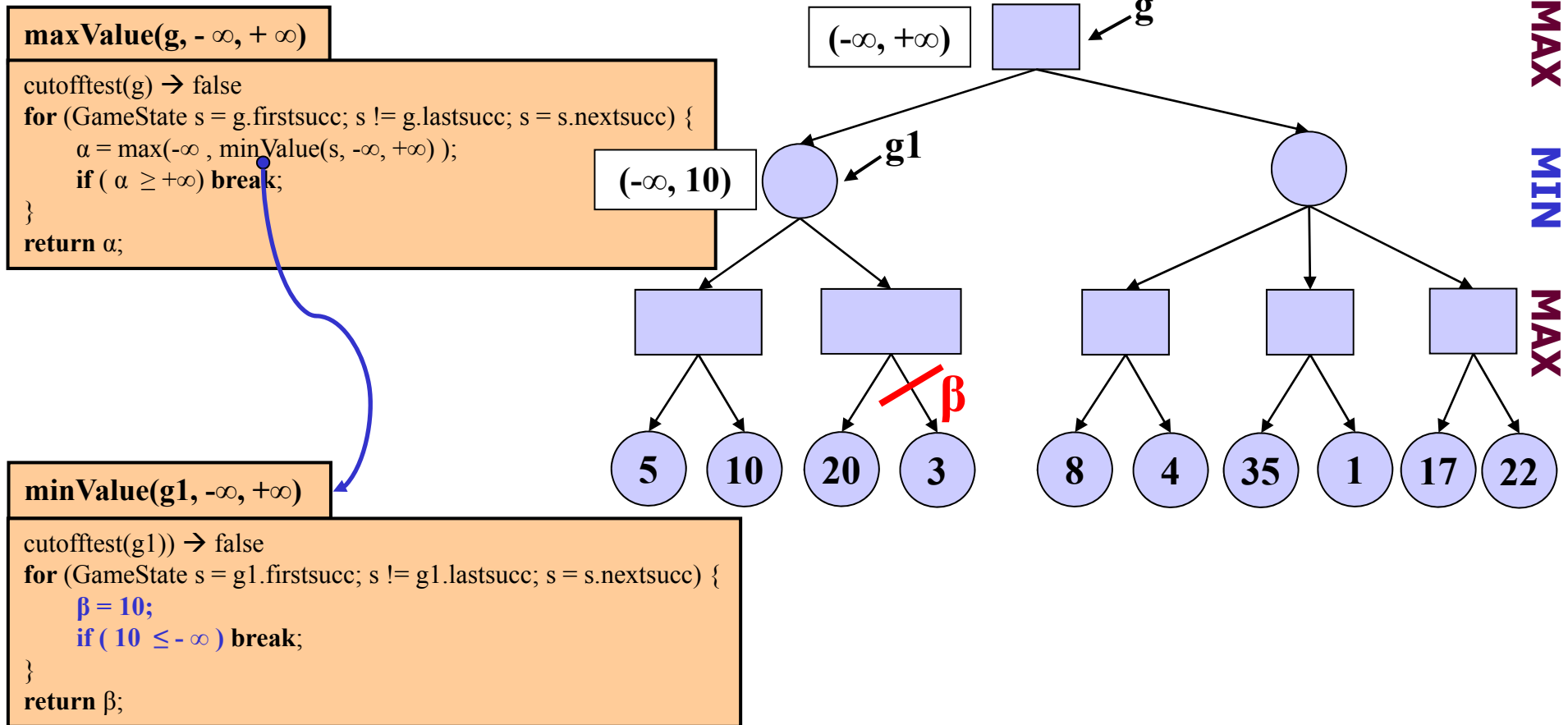










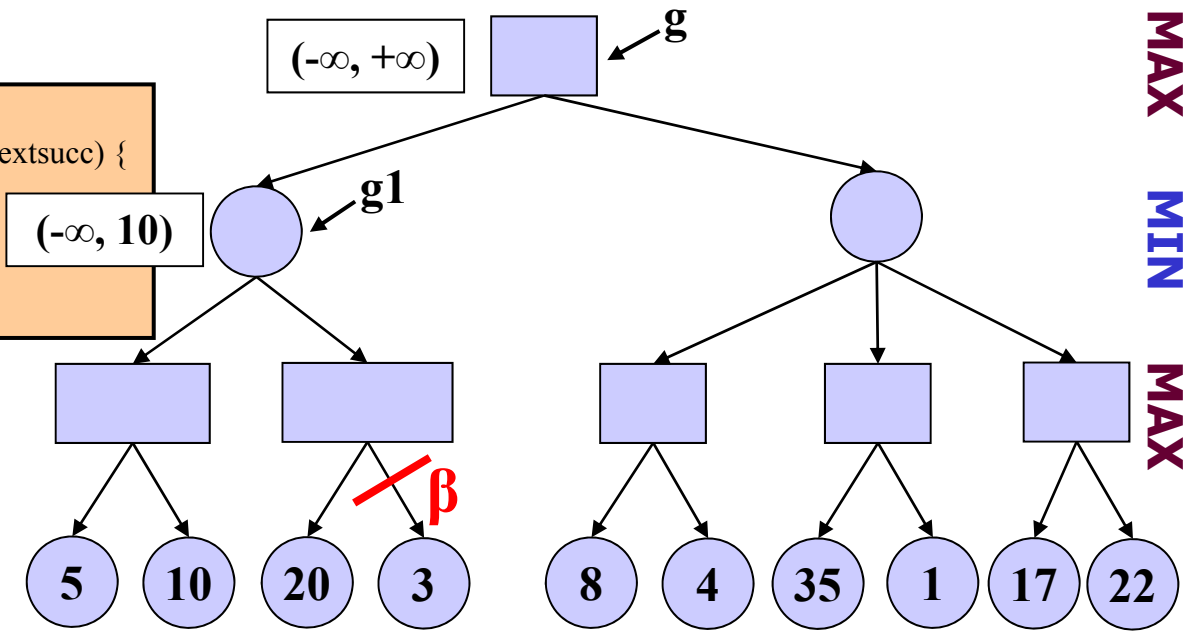


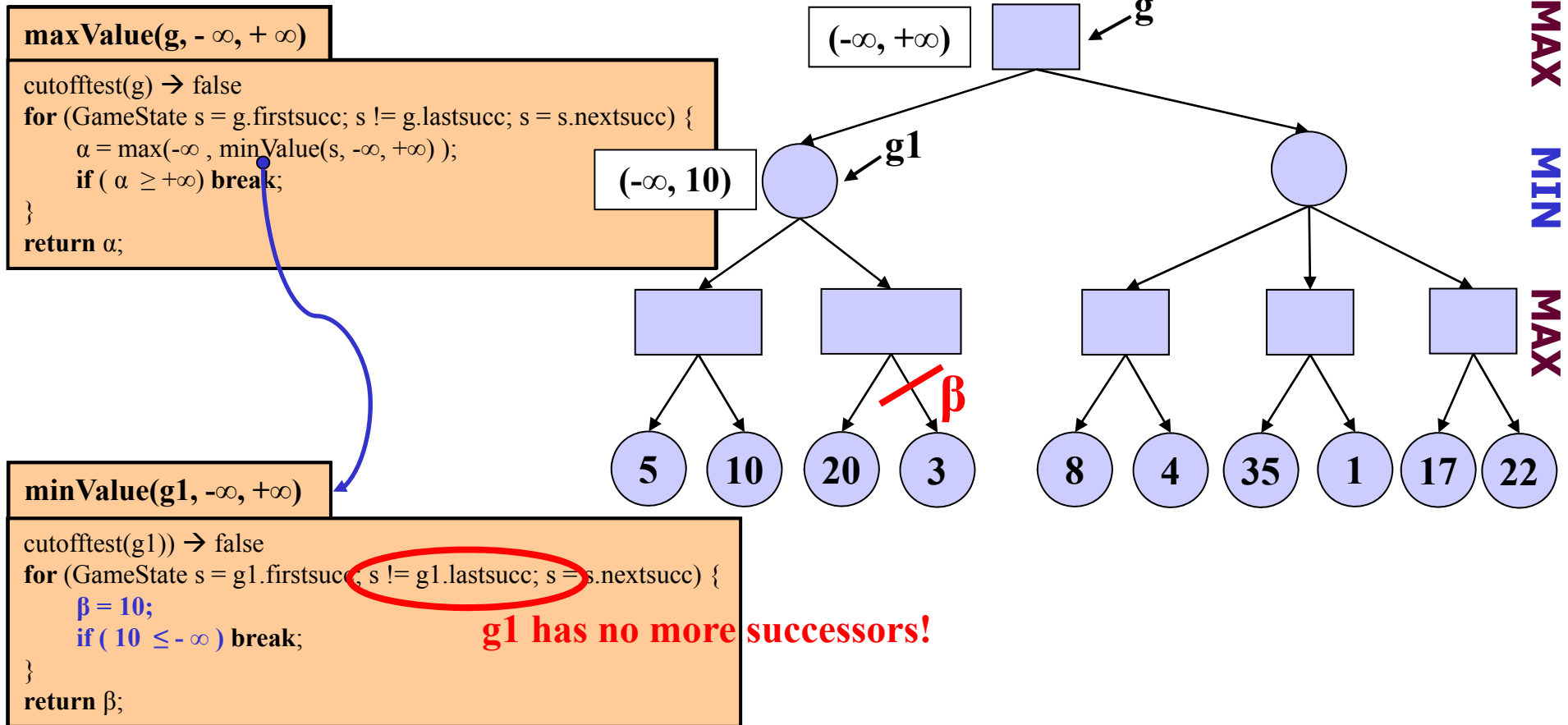
```
maxValue(g, -∞, +∞)
```

```
  cutofftest(g) → false  
  for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {  
    α = max(-∞, minValue(s, -∞, +∞));  
    if (α ≥ +∞) break;  
  }  
  return α;
```

```
minValue(g1, -∞, +∞)
```

```
  cutofftest(g1) → false  
  for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {  
    β = 10;  
    if (10 ≤ -∞) break;  
  }  
  return β; false!
```



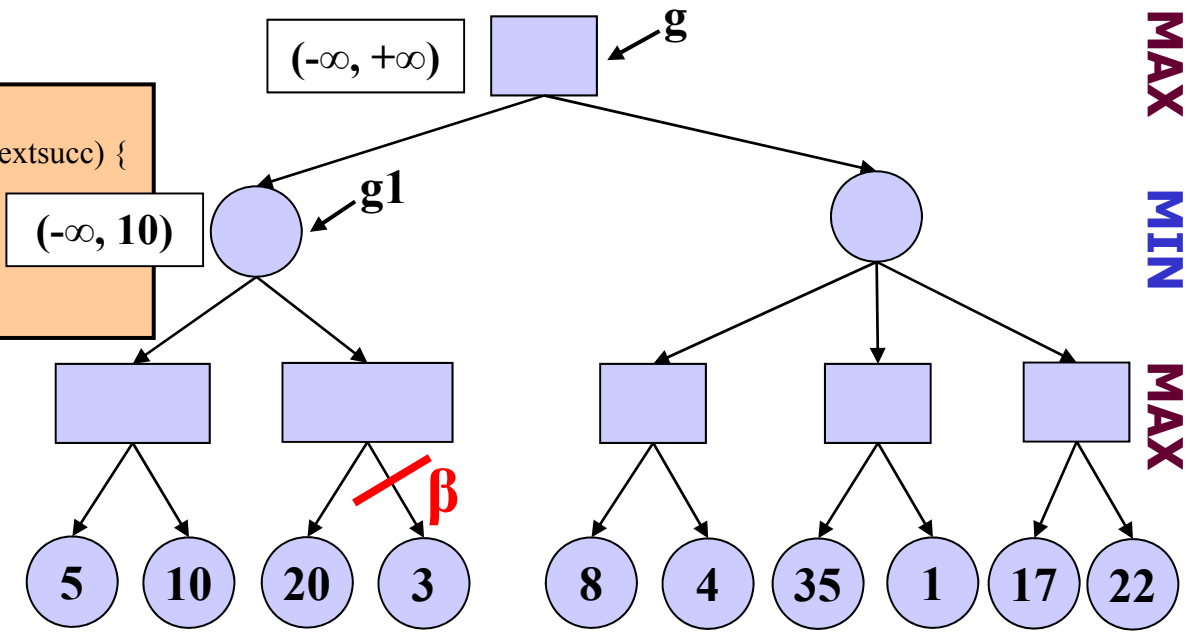


```
maxValue(g, -∞, +∞)
```

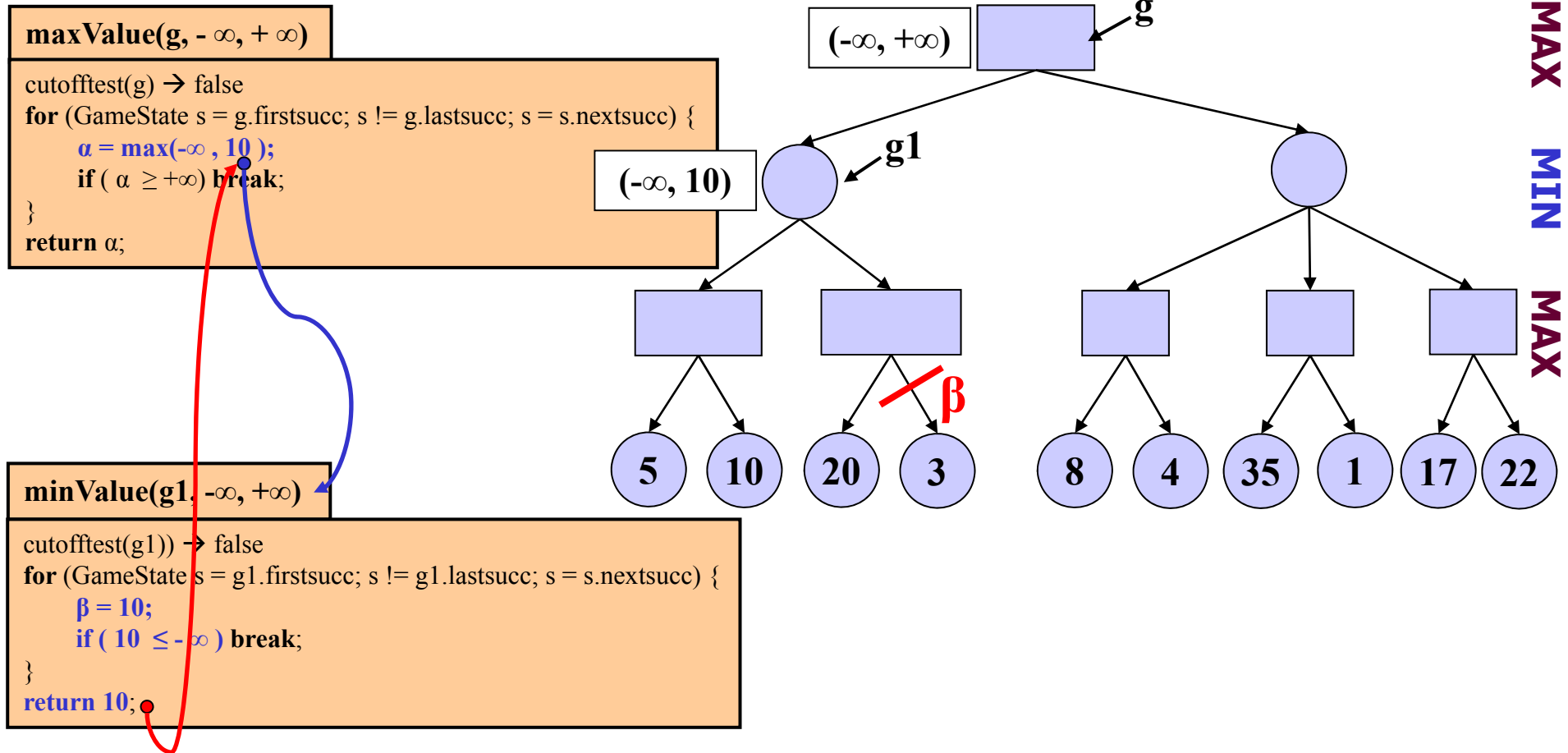
```
cutofftest(g) → false  
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {  
    α = max(-∞, minValue(s, -∞, +∞));  
    if (α ≥ +∞) break;  
}  
return α;
```

```
minValue(g1, -∞, +∞)
```

```
cutofftest(g1) → false  
for (GameState s = g1.firstsucc, s != g1.lastsucc; s = s.nextsucc) {  
    β = 10;  
    if (10 ≤ -∞) break;  
}  
return 10;
```



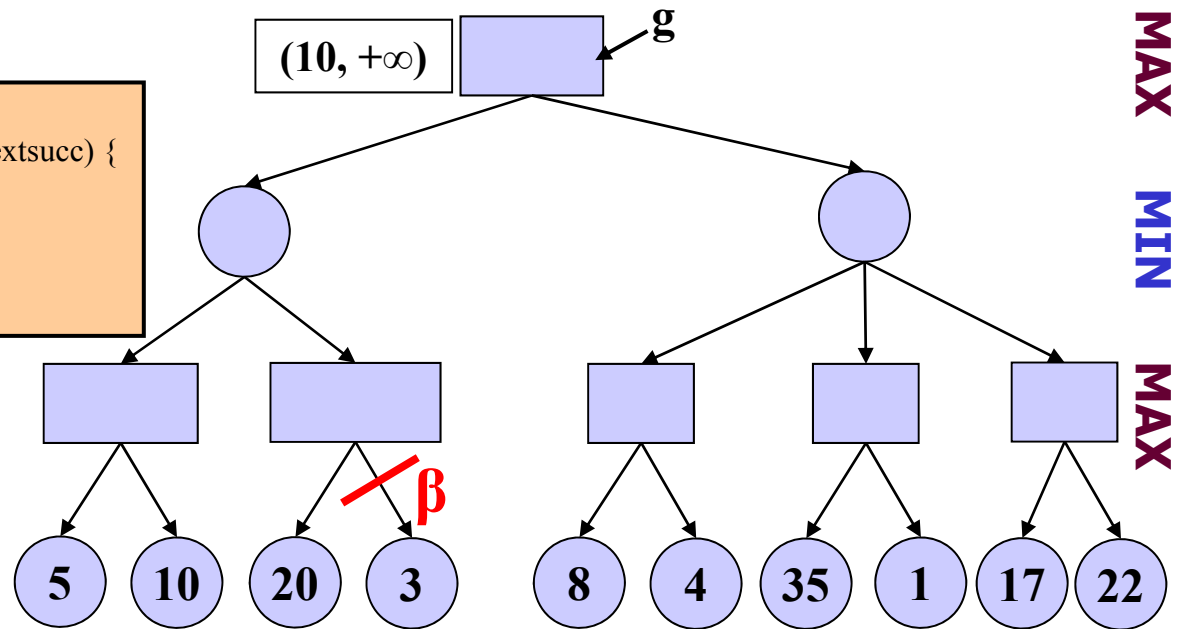
**g1 has no more successors!**



```

maxValue(g, -∞, +∞)
cutofftest(g) → false
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {
  α = 10;
  if ( α ≥ +∞) break;
}
return α;

```

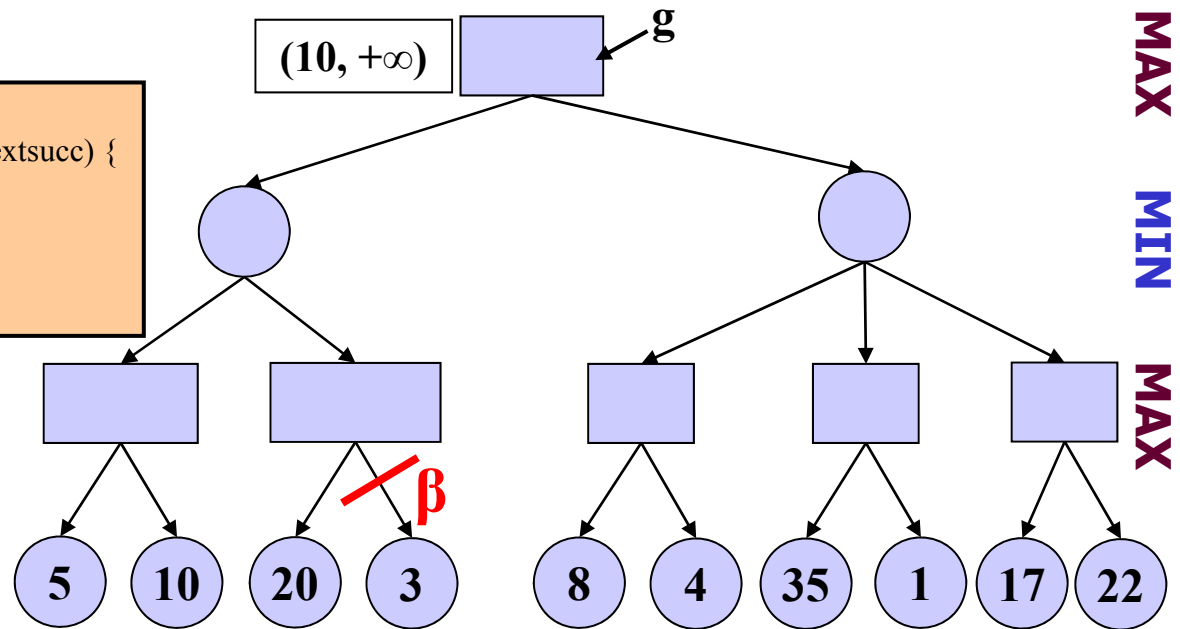


```

maxValue(g, -∞, +∞)
cutofftest(g) → false
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {
   $\alpha = 10$ ;
  if (  $10 \geq +\infty$  ) break;
}
return  $\alpha$ ;

```

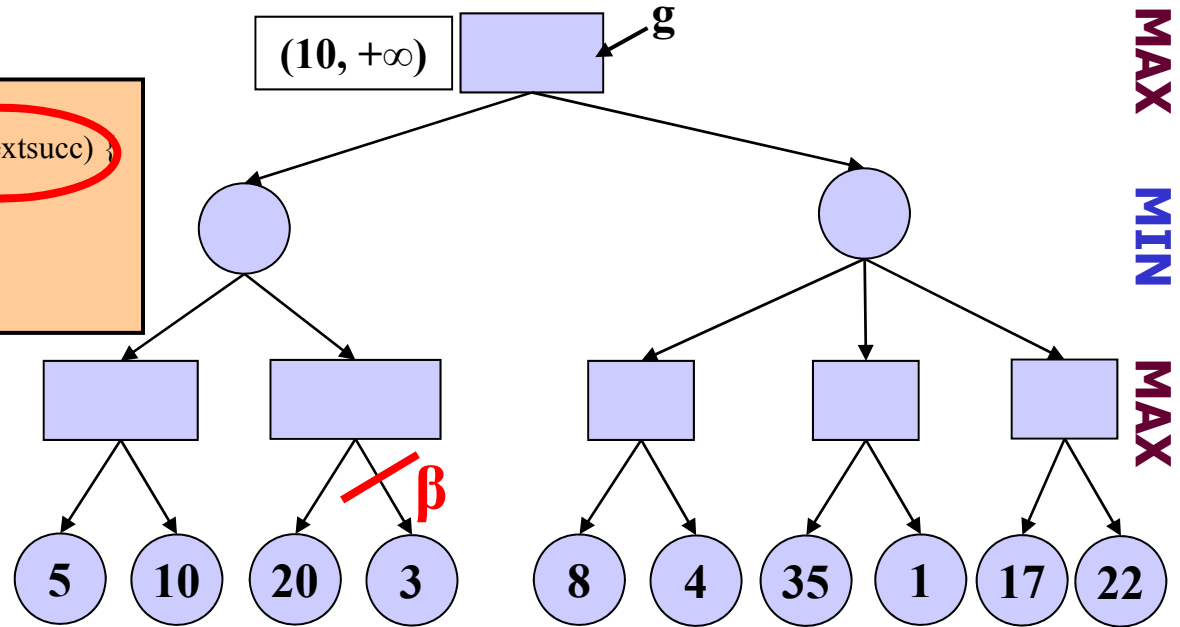
**false!**





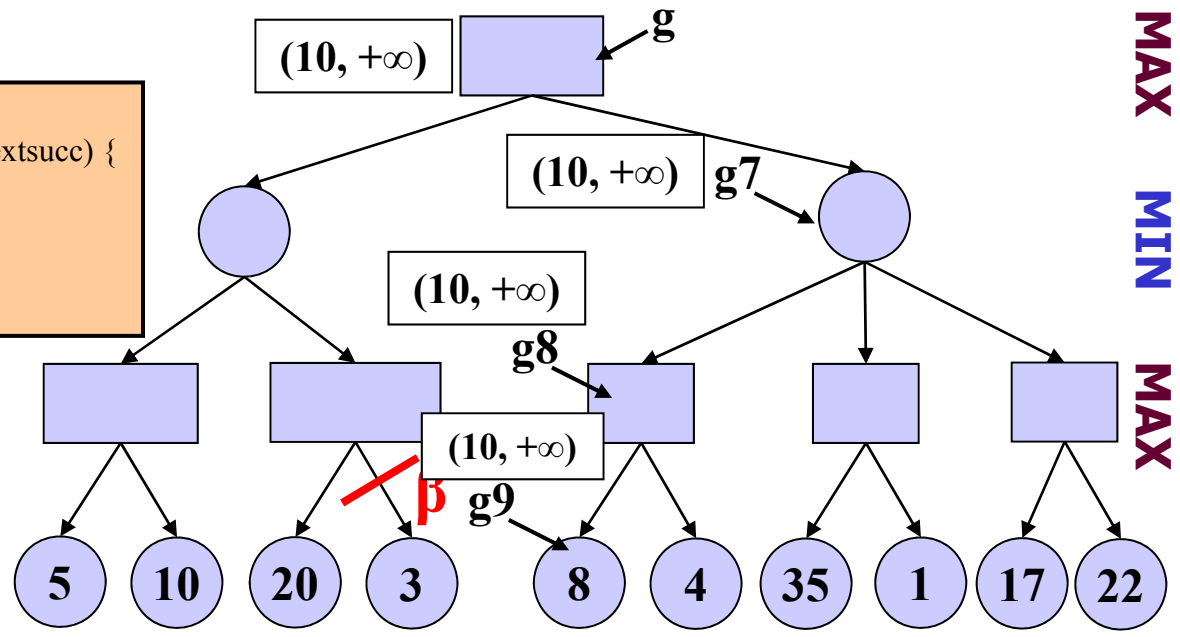
**maxValue(g, -∞, +∞)**

```
cutofftest(g) → false  
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {  
    α = max(10, minValue(s, 10, +∞));  
    if (α ≥ +∞) break;  
}  
return α;
```



**maxValue(g, -∞, +∞)**

```
cutofftest(g) → false
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {
  α = max(10, minValue(s, 10, +∞));
  if (α ≥ +∞) break;
}
return α;
```



**minValue(g7, 10, +∞)**

```
cutofftest(g7) → false
for (GameState s = g7.firstsucc; s != g7.lastsucc; s = s.nextsucc) {
  β = min(+∞, maxValue(s, 10, +∞));
  if (β ≤ 10) break;
}
return β;
```

**maxValue(g8, 10, +∞)**

```
cutofftest(g8) → false
for (GameState s = g8.firstsucc; s != g8.lastsucc; s = s.nextsucc) {
  α = max(10, minValue(s, 10, +∞));
  if (α ≥ +∞) break;
}
return α;
```

**minValue(g9, 10, +∞)**

```
minValue(g9, 10, +∞)
```

```
maxValue(g, -∞, +∞)
```

```
cutfftest(g) → false  
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {  
  α = max(10, minValue(s, 10, +∞));  
  if (α ≥ +∞) break;  
}  
return α;
```

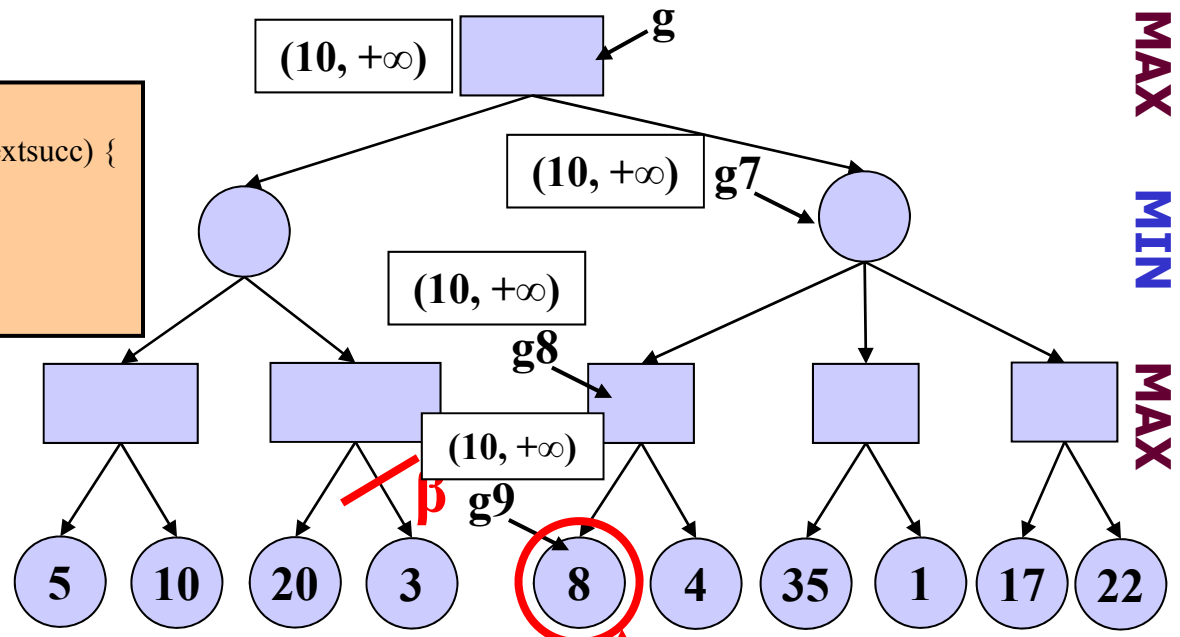
```
minValue(g7, 10, +∞)
```

```
cutfftest(g7) → false  
for (GameState s = g7.firstsucc; s != g7.lastsucc; s = s.nextsucc) {  
  β = min(+∞, maxValue(s, 10, +∞));  
  if (β ≤ 10) break;  
}  
return β;
```

```
maxValue(g8, 10, +∞)
```

```
cutfftest(g8) → false  
for (GameState s = g8.firstsucc; s != g8.lastsucc; s = s.nextsucc) {  
  α = max(10, minValue(s, 10, +∞));  
  if (α ≥ +∞) break;  
}  
return α;
```

```
minValue(g9, 10, +∞)  
cutfftest(g9) → true!!
```



```
maxValue(g, -∞, +∞)
```

```
cutfftest(g) → false  
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {  
    α = max(10, minValue(s, 10, +∞));  
    if (α ≥ +∞) break;  
}  
return α;
```

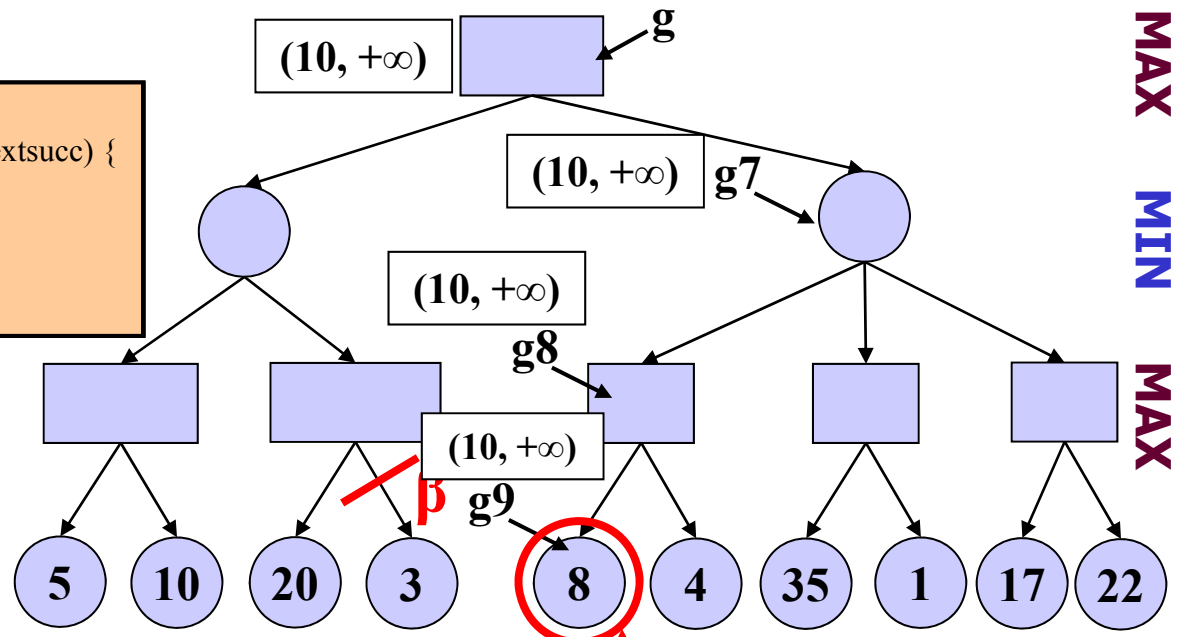
```
minValue(g7, 10, +∞)
```

```
cutfftest(g7) → false  
for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {  
    β = min(+∞, maxValue(s, 10, +∞));  
    if (β ≤ 10) break;  
}  
return β;
```

```
maxValue(g8, 10, +∞)
```

```
cutfftest(g8) → false  
for (GameState s = g8.firstsucc; s != g8.lastsucc; s = s.nextsucc) {  
    α = max(10, minValue(s, 10, +∞));  
    if (α ≥ +∞) break;  
}  
return α;
```

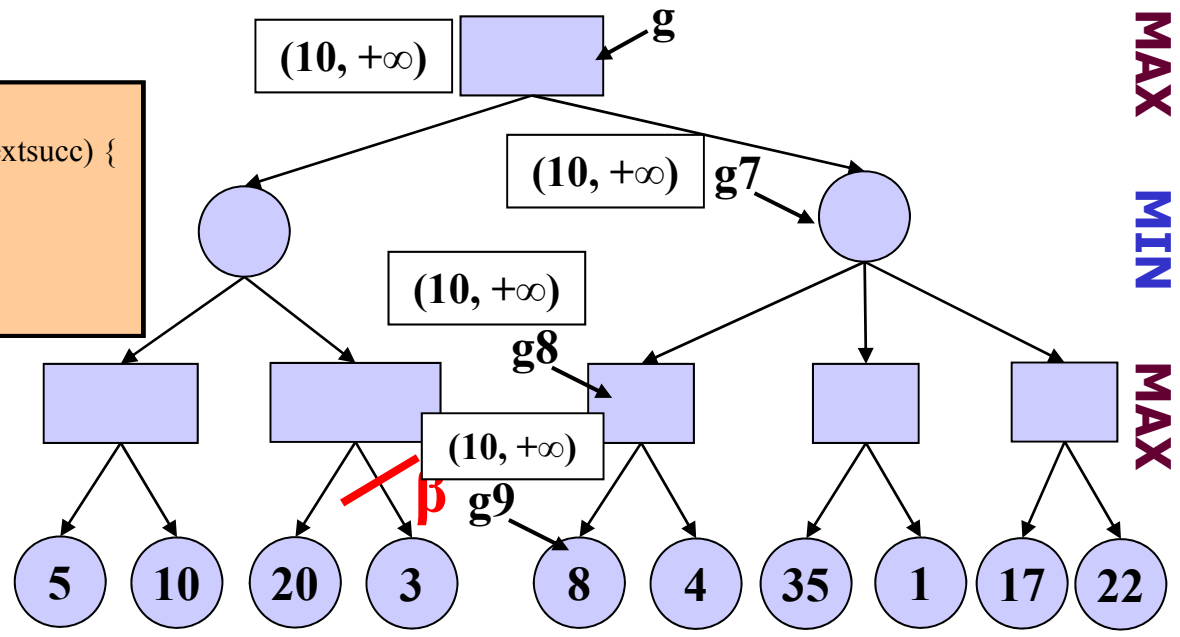
```
minValue(g9, 10, +∞)  
cutfftest(g9) → true!!  
⇒ eval(g9) = 8  
⇒ return 8
```



MAX  
MIN  
MAX

**maxValue(g, -∞, +∞)**

```
cutofftest(g) → false  
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {  
    α = max(10, minValue(s, 10, +∞));  
    if (α ≥ +∞) break;  
}  
return α;
```



**minValue(g7, 10, +∞)**

```
cutofftest(g7) → false  
for (GameState s = g7.firstsucc; s != g7.lastsucc; s = s.nextsucc) {  
    β = min(+∞, maxValue(s, 10, +∞));  
    if (β ≤ 10) break;  
}  
return β;
```

**maxValue(g8, 10, +∞)**

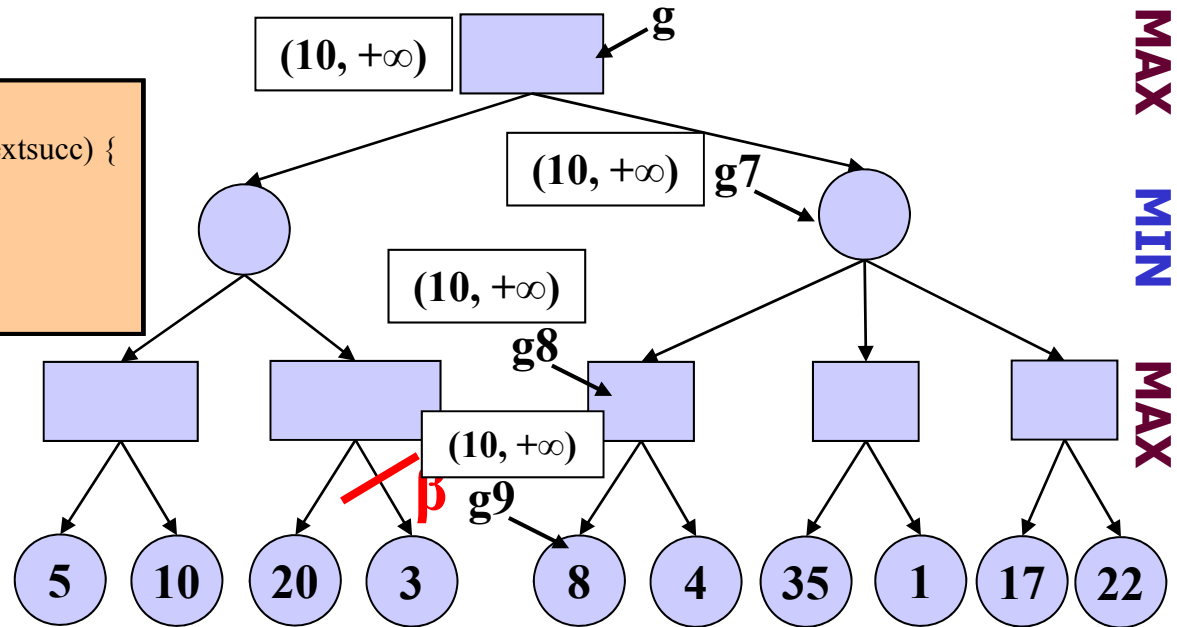
```
cutofftest(g8) → false  
for (GameState s = g8.firstsucc; s != g8.lastsucc; s = s.nextsucc) {  
    α = max(10, minValue(s, 10, +∞));  
    if (α ≥ +∞) break;  
}  
return α;
```

**minValue(g9, 10, +∞)**

```
cutofftest(g9) → true!!  
⇒ eval(g9) = 8  
⇒ return 8
```

```
maxValue(g, -∞, +∞)
```

```
cutofftest(g) → false  
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {  
    α = max(10, minValue(s, 10, +∞));  
    if (α ≥ +∞) break;  
}  
return α;
```



```
minValue(g7, 10, +∞)
```

```
cutofftest(g7) → false  
for (GameState s = g7.firstsucc; s != g7.lastsucc; s = s.nextsucc) {  
    β = min(+∞, maxValue(s, 10, +∞));  
    if (β ≤ 10) break;  
}  
return β;
```

```
maxValue(g8, 10, +∞)
```

```
cutofftest(g8) → false  
for (GameState s = g8.firstsucc; s != g8.lastsucc; s = s.nextsucc) {  
    α = max(10, 8);  
    if (α ≥ +∞) break;  
}  
return α;
```

```
minValue(g9, 10, +∞)
```

```
cutofftest(g9) → true!!  
⇒ eval(g9) = 8  
⇒ return 8
```

**maxValue(g, -∞, +∞)**

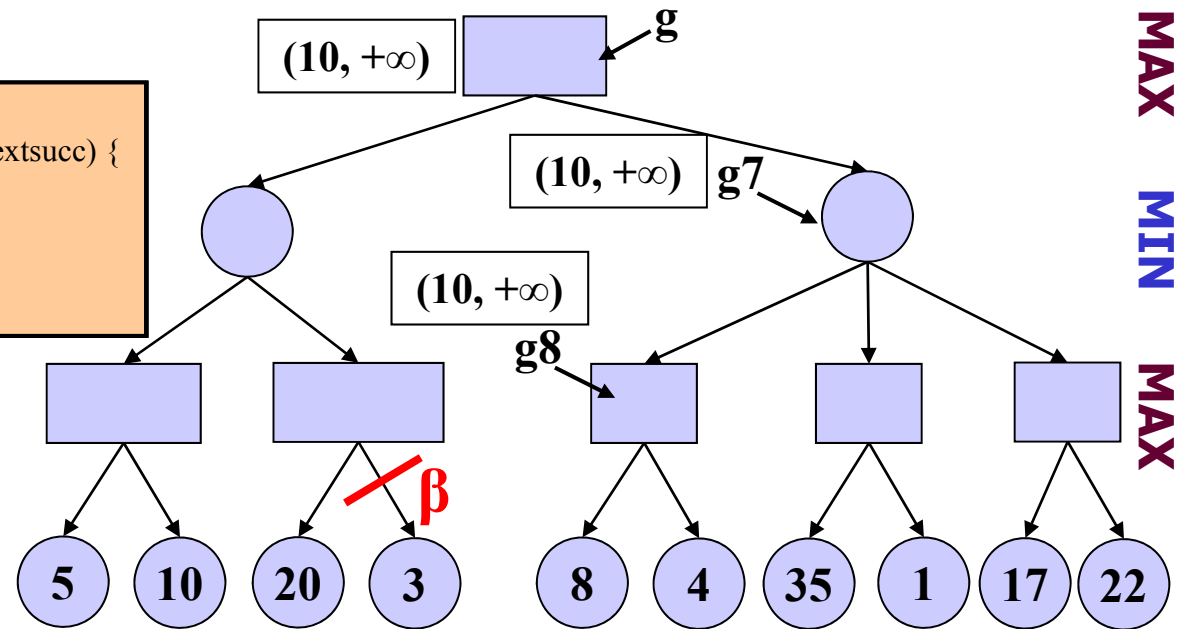
```
cutfftest(g) → false  
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {  
    α = max(10, minValue(s, 10, +∞));  
    if (α ≥ +∞) break;  
}  
return α;
```

**minValue(g7, 10, +∞)**

```
cutfftest(g7) → false  
for (GameState s = g7.firstsucc; s != g7.lastsucc; s = s.nextsucc) {  
    β = min(+∞, maxValue(s, 10, +∞));  
    if (β ≤ 10) break;  
}  
return β;
```

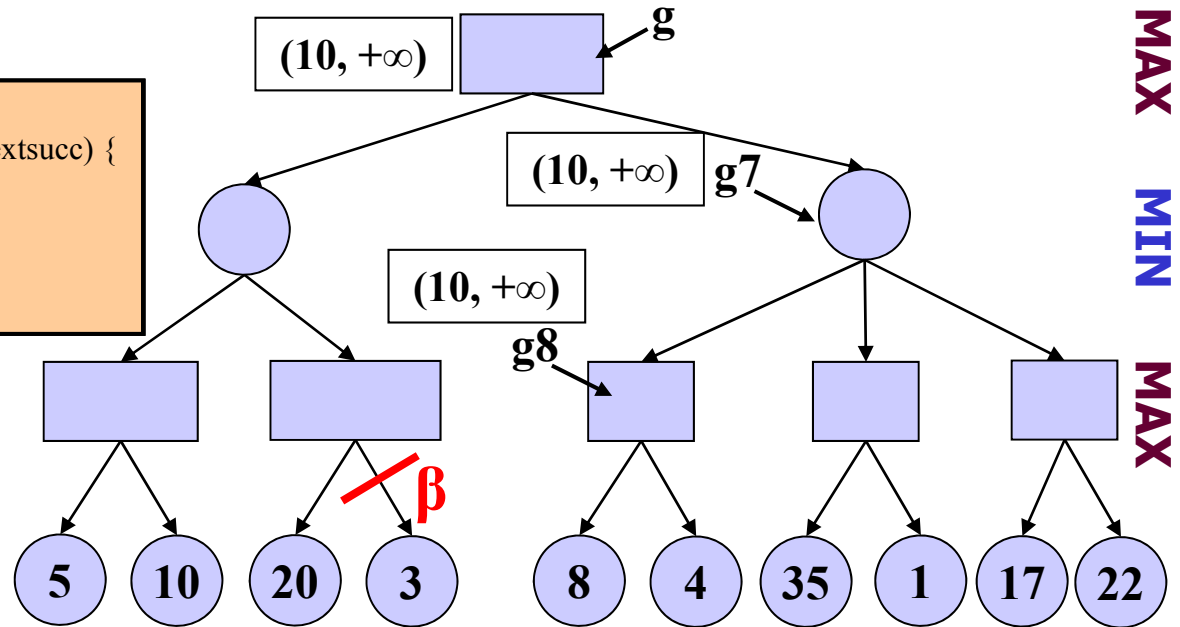
**maxValue(g8, 10, +∞)**

```
cutfftest(g8) → false  
for (GameState s = g8.firstsucc; s != g8.lastsucc; s = s.nextsucc) {  
    α = 10;  
    if (α ≥ +∞) break;  
}  
return α;
```



**maxValue(g, -∞, +∞)**

```
cutofftest(g) → false
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {
  α = max(10, minValue(s, 10, +∞));
  if (α ≥ +∞) break;
}
return α;
```



**minValue(g7, 10, +∞)**

```
cutofftest(g7) → false
for (GameState s = g7.firstsucc; s != g7.lastsucc; s = s.nextsucc) {
  β = min(+∞, maxValue(s, 10, +∞));
  if (β ≤ 10) break;
}
return β;
```

**maxValue(g8, 10, +∞)**

```
cutofftest(g8) → false
for (GameState s = g8.firstsucc; s != g8.lastsucc; s = s.nextsucc) {
  α = 10
  if (10 ≥ +∞) break;
}
return α; false!
```



**maxValue(g, -∞, +∞)**

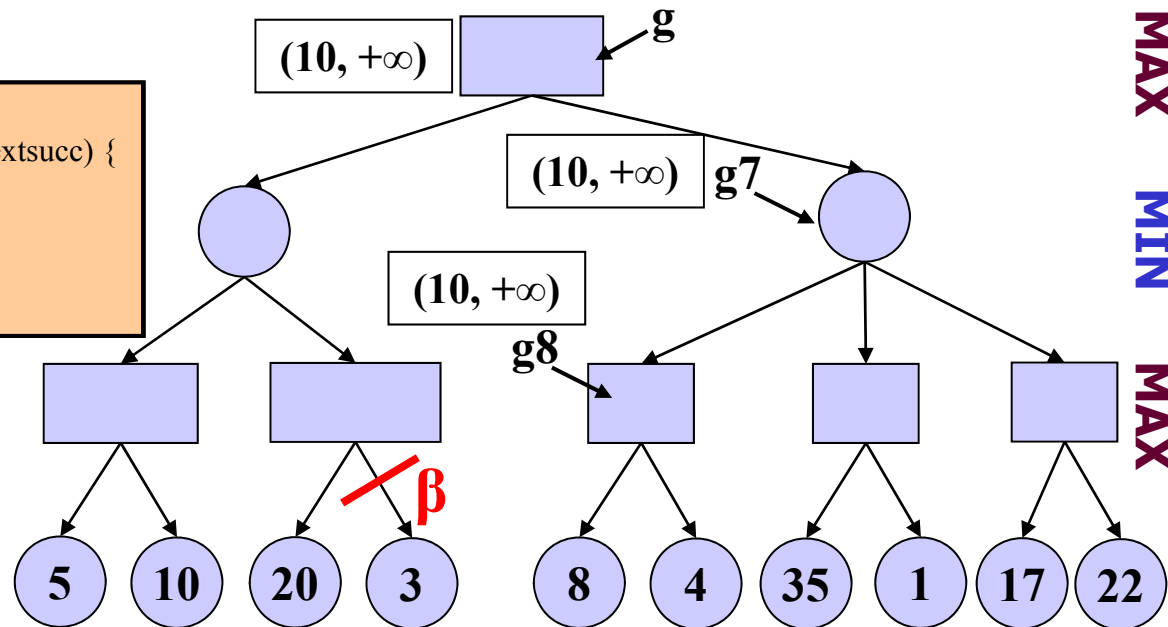
```
cutofftest(g) → false  
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {  
    α = max(10, minValue(s, 10, +∞));  
    if (α ≥ +∞) break;  
}  
return α;
```

**minValue(g7, 10, +∞)**

```
cutofftest(g7) → false  
for (GameState s = g7.firstsucc; s != g7.lastsucc; s = s.nextsucc) {  
    β = min(+∞, maxValue(s, 10, +∞));  
    if (β ≤ 10) break;  
}  
return β;
```

**maxValue(g8, 10, +∞)**

```
cutofftest(g8) → false  
for (GameState s = g8.firstsucc; s != g8.lastsucc; s = s.nextsucc) {  
    α = max(10, minValue(s, 10, +∞));  
    if (α ≥ +∞) break;  
}  
return α;
```



next successor

**maxValue(g, -∞, +∞)**

```
cutofftest(g) → false  
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {  
    α = max(10, minValue(s, 10, +∞));  
    if (α ≥ +∞) break;  
}  
return α;
```

**minValue(g7, 10, +∞)**

```
cutofftest(g7) → false  
for (GameState s = g7.firstsucc; s != g7.lastsucc; s = s.nextsucc) {  
    β = min(+∞, maxValue(s, 10, +∞));  
    if (β ≤ 10) break;  
}  
return β;
```

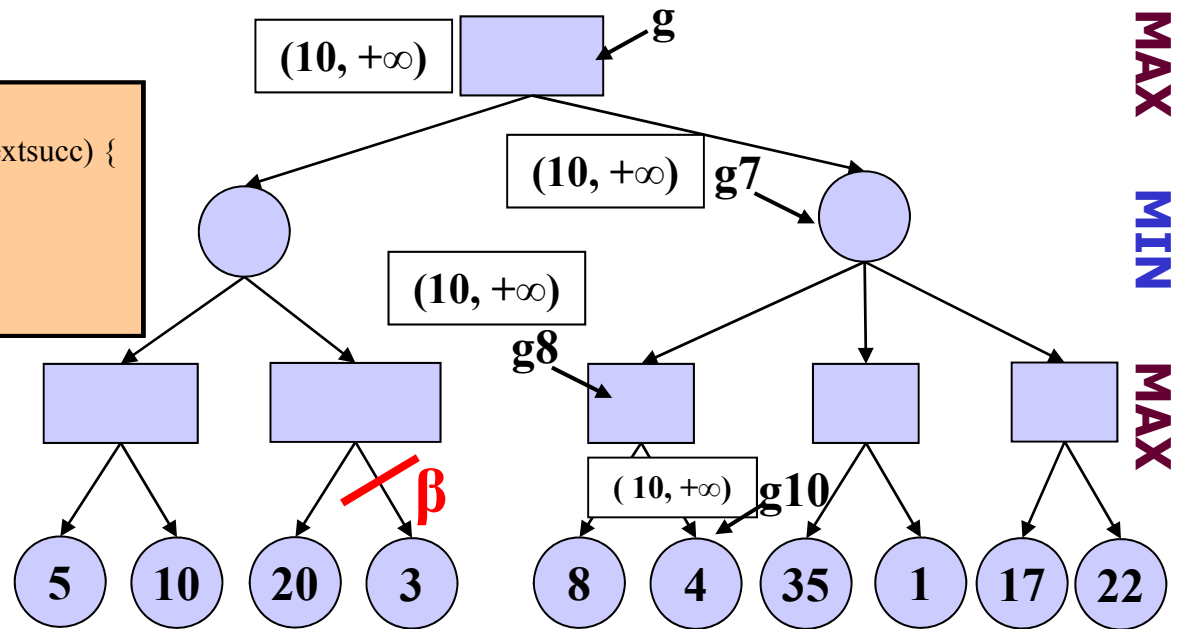
**maxValue(g8, 10, +∞)**

```
cutofftest(g8) → false  
for (GameState s = g8.firstsucc; s != g8.lastsucc; s = s.nextsucc) {  
    α = max(10, minValue(s, 10, +∞));  
    if (α ≥ +∞) break;  
}  
return α;
```

**minValue(g10, 10, +∞)**

```
  

```



**maxValue(g, -∞, +∞)**

```
cutfftest(g) → false  
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {  
    α = max(10, minValue(s, 10, +∞));  
    if (α ≥ +∞) break;  
}  
return α;
```

**minValue(g7, 10, +∞)**

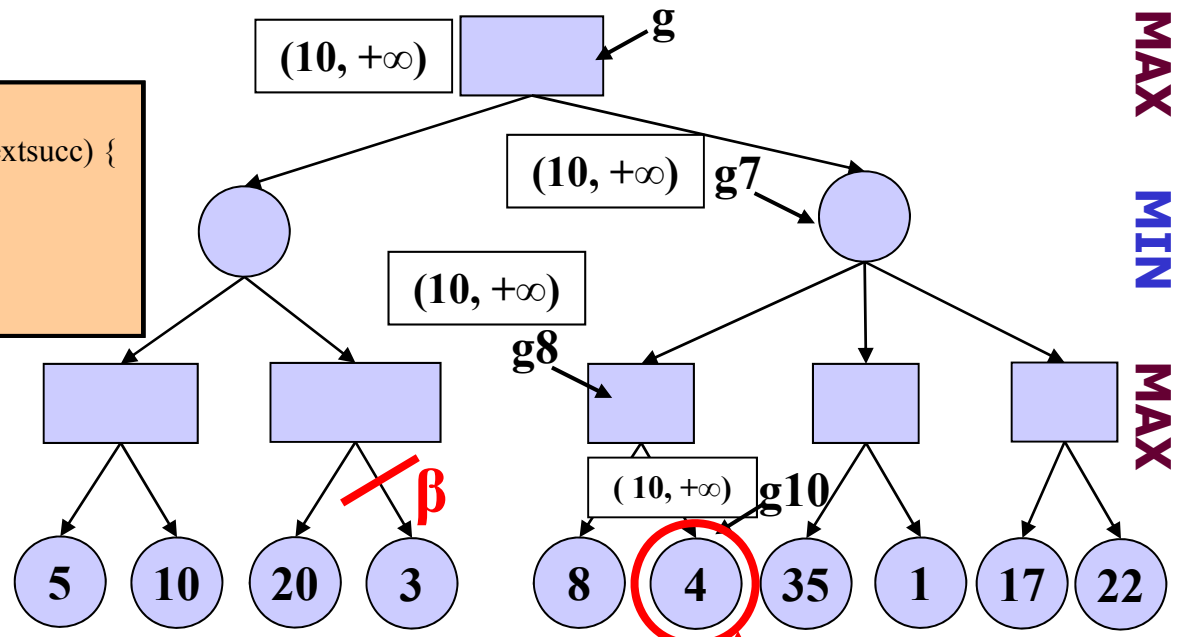
```
cutfftest(g7) → false  
for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {  
    β = min(+∞, maxValue(s, 10, +∞));  
    if (β ≤ 10) break;  
}  
return β;
```

**maxValue(g8, 10, +∞)**

```
cutfftest(g8) → false  
for (GameState s = g8.firstsucc; s != g8.lastsucc; s = s.nextsucc) {  
    α = max(10, minValue(s, 10, +∞));  
    if (α ≥ +∞) break;  
}  
return α;
```

**minValue(g10, 10, +∞)**

```
cutfftest(g10) → true!!
```



**maxValue(g, -∞, +∞)**

```
cutfftest(g) → false  
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {  
    α = max(10, minValue(s, 10, +∞));  
    if (α ≥ +∞) break;  
}  
return α;
```

**minValue(g7, 10, +∞)**

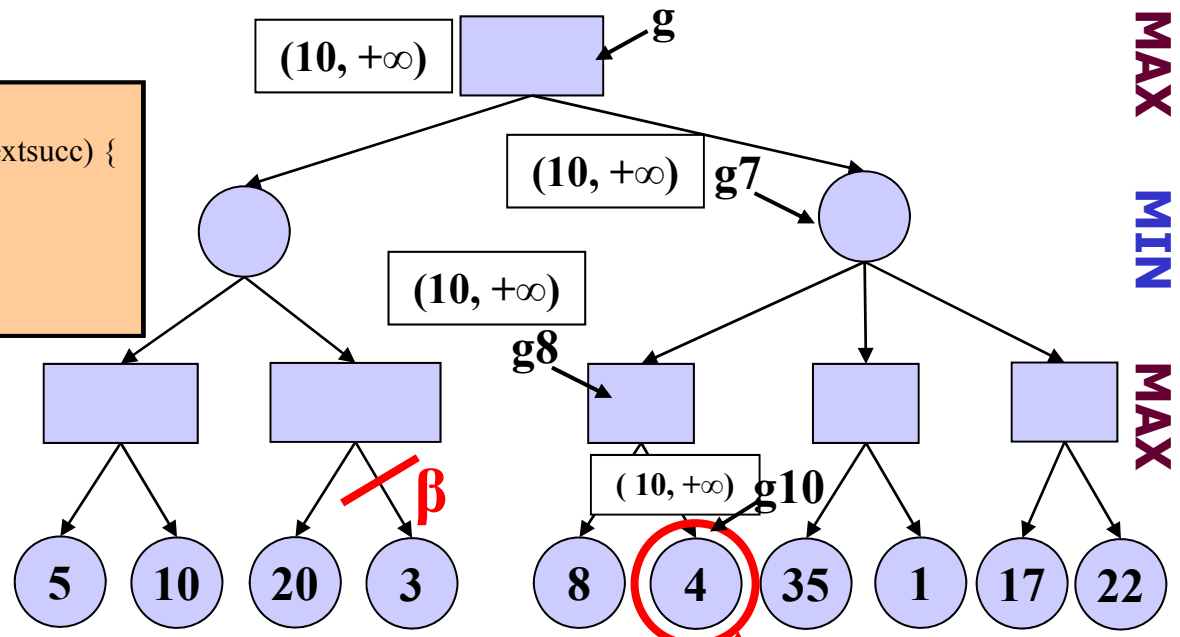
```
cutfftest(g7) → false  
for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {  
    β = min(+∞, maxValue(s, 10, +∞));  
    if (β ≤ 10) break;  
}  
return β;
```

**maxValue(g8, 10, +∞)**

```
cutfftest(g8) → false  
for (GameState s = g8.firstsucc; s != g8.lastsucc; s = s.nextsucc) {  
    α = max(10, minValue(s, 10, +∞));  
    if (α ≥ +∞) break;  
}  
return α;
```

**minValue(g10, 10, +∞)**

```
cutfftest(g10) → true!!  
eval(g10) = 4  
return 4
```



**maxValue(g, -∞, +∞)**

```
cutfftest(g) → false
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {
  α = max(10, minValue(s, 10, +∞));
  if (α ≥ +∞) break;
}
return α;
```

**minValue(g7, 10, +∞)**

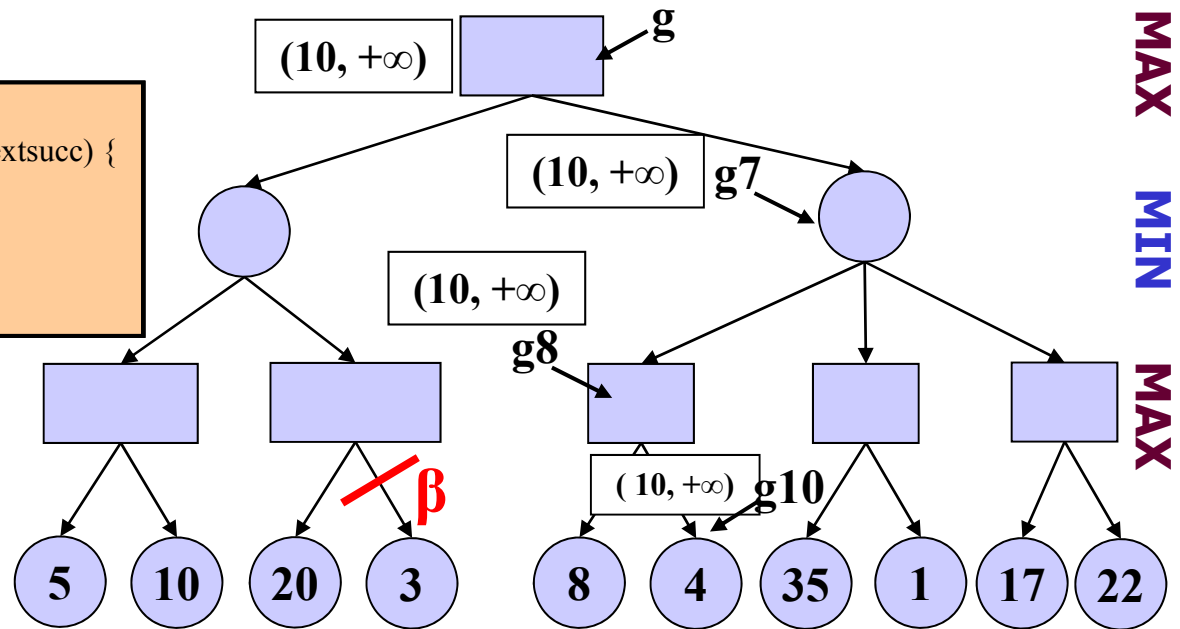
```
cutfftest(g7) → false
for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {
  β = min(+∞, maxValue(s, 10, +∞));
  if (β ≤ 10) break;
}
return β;
```

**maxValue(g8, 10, +∞)**

```
cutfftest(g8) → false
for (GameState s = g8.firstsucc; s != g8.lastsucc; s = s.nextsucc) {
  α = max(10, minValue(s, 10, +∞));
  if (α ≥ +∞) break;
}
return α;
```

**minValue(g10, 10, +∞)**

```
cutfftest(g10) → true!!
eval(g10) = 4
return 4
```



**maxValue(g, -∞, +∞)**

```
cutfftest(g) → false
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {
  α = max(10, minValue(s, 10, +∞));
  if (α ≥ +∞) break;
}
return α;
```

**minValue(g7, 10, +∞)**

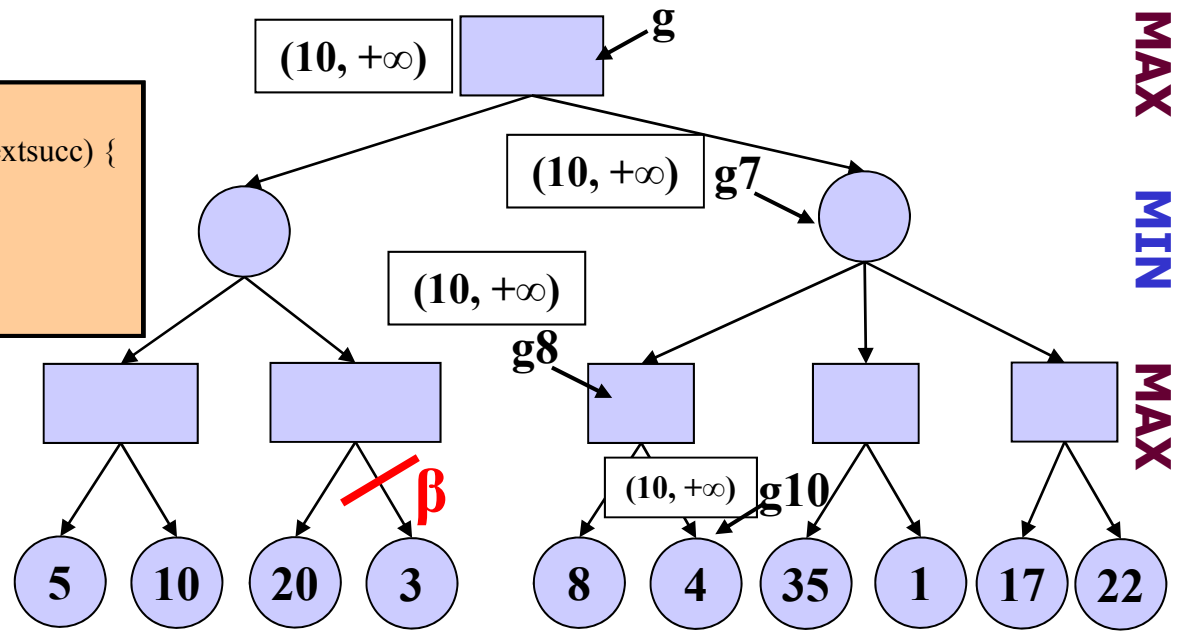
```
cutfftest(g7) → false
for (GameState s = g7.firstsucc; s != g7.lastsucc; s = s.nextsucc) {
  β = min(+∞, maxValue(s, 10, +∞));
  if (β ≤ 10) break;
}
return β;
```

**maxValue(g8, 10, +∞)**

```
cutfftest(g8) → false
for (GameState s = g8.firstsucc; s != g8.lastsucc; s = s.nextsucc) {
  α = max(10, 4);
  if (α ≥ +∞) break;
}
return α;
```

**minValue(g10, 10, +∞)**

```
cutfftest(g10) → true!!
eval(g10) = 4
return 4
```



**maxValue(g, -∞, +∞)**

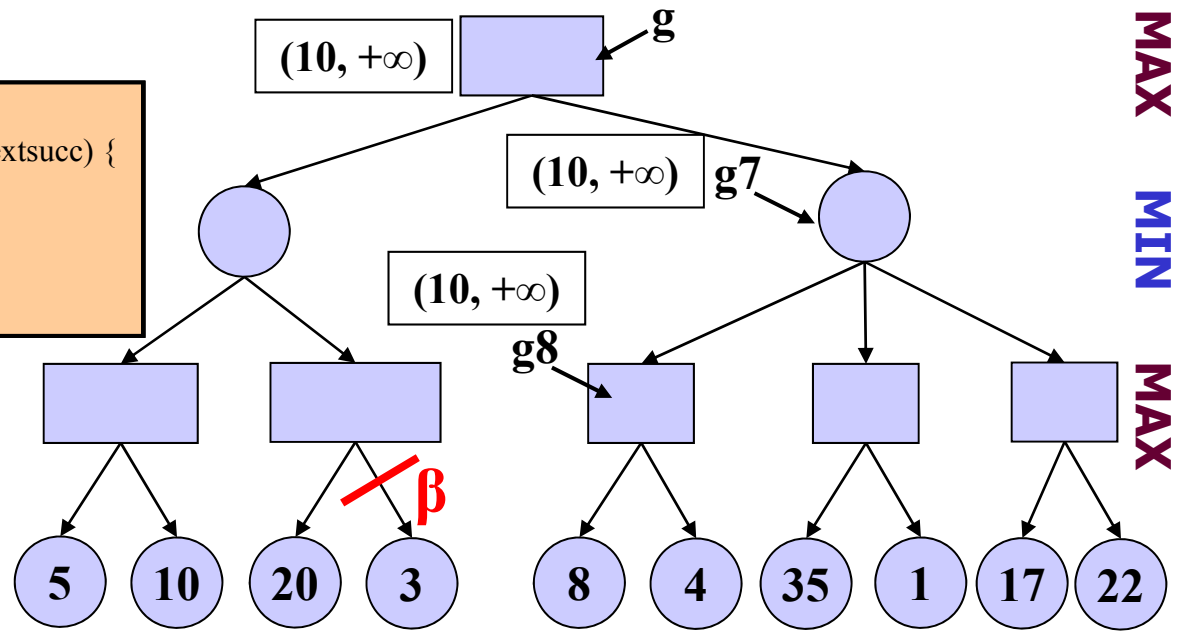
```
cutofftest(g) → false  
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {  
    α = max(10, minValue(s, 10, +∞));  
    if (α ≥ +∞) break;  
}  
return α;
```

**minValue(g7, 10, +∞)**

```
cutofftest(g7) → false  
for (GameState s = g7.firstsucc; s != g7.lastsucc; s = s.nextsucc) {  
    β = min(+∞, maxValue(s, 10, +∞));  
    if (β ≤ 10) break;  
}  
return β;
```

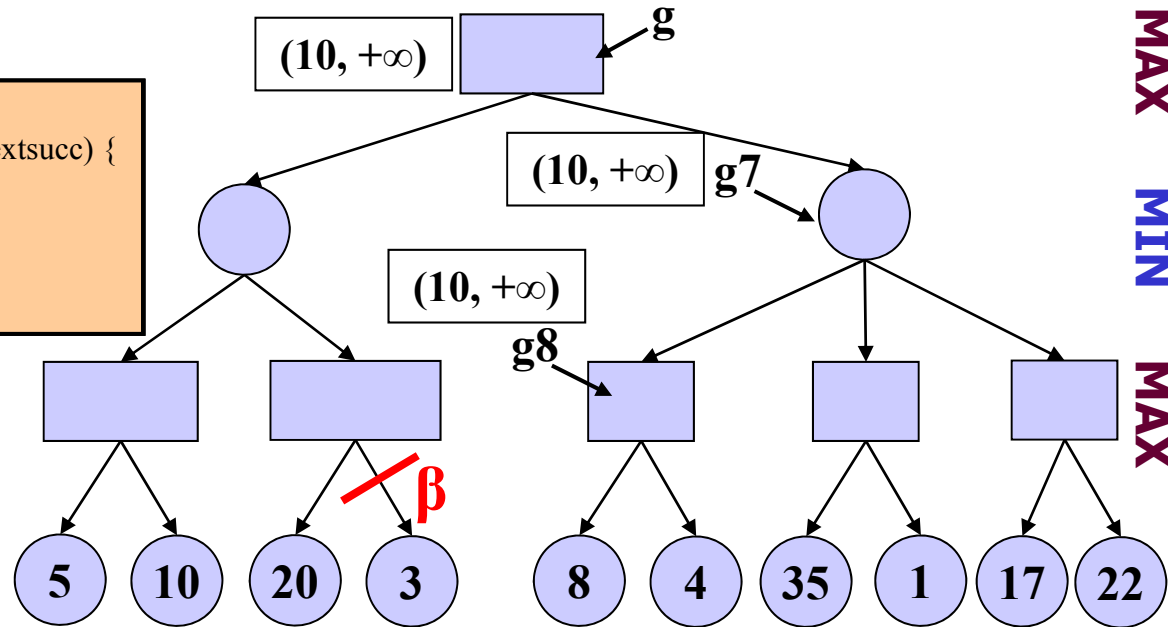
**maxValue(g8, 10, +∞)**

```
cutofftest(g8) → false  
for (GameState s = g8.firstsucc; s != g8.lastsucc; s = s.nextsucc) {  
    α = 10;  
    if (α ≥ +∞) break;  
}  
return α;
```



MAX  
MIN  
MAX

```
maxValue(g, -∞, +∞)  
cutofftest(g) → false  
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {  
    α = max(10, minValue(s, 10, +∞));  
    if (α ≥ +∞) break;  
}  
return α;
```



```
minValue(g7, 10, +∞)  
cutofftest(g7) → false  
for (GameState s = g7.firstsucc; s != g7.lastsucc; s = s.nextsucc) {  
    β = min(+∞, maxValue(s, 10, +∞));  
    if (β ≤ 10) break;  
}  
return β;
```

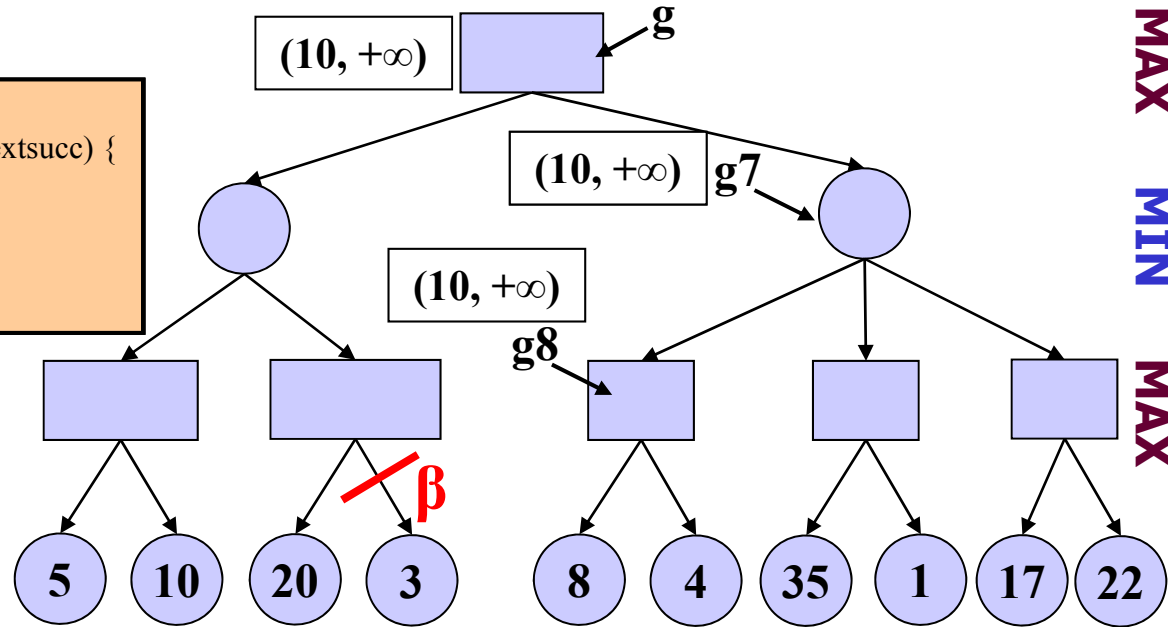
```
maxValue(g8, 10, +∞)  
cutofftest(g8) → false  
for (GameState s = g8.firstsucc; s != g8.lastsucc; s = s.nextsucc) {  
    α = 10;  
    if (10 ≥ +∞) break;  
}  
return α, false!
```



MAX  
MIN  
MAX

**maxValue(g, -∞, +∞)**

```
cutofftest(g) → false  
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {  
    α = max(10, minValue(s, 10, +∞));  
    if (α ≥ +∞) break;  
}  
return α;
```



**minValue(g7, 10, +∞)**

```
cutofftest(g7) → false  
for (GameState s = g7.firstsucc; s != g7.lastsucc; s = s.nextsucc) {  
    β = min(+∞, maxValue(s, 10, +∞));  
    if (β ≤ 10) break;  
}  
return β;
```

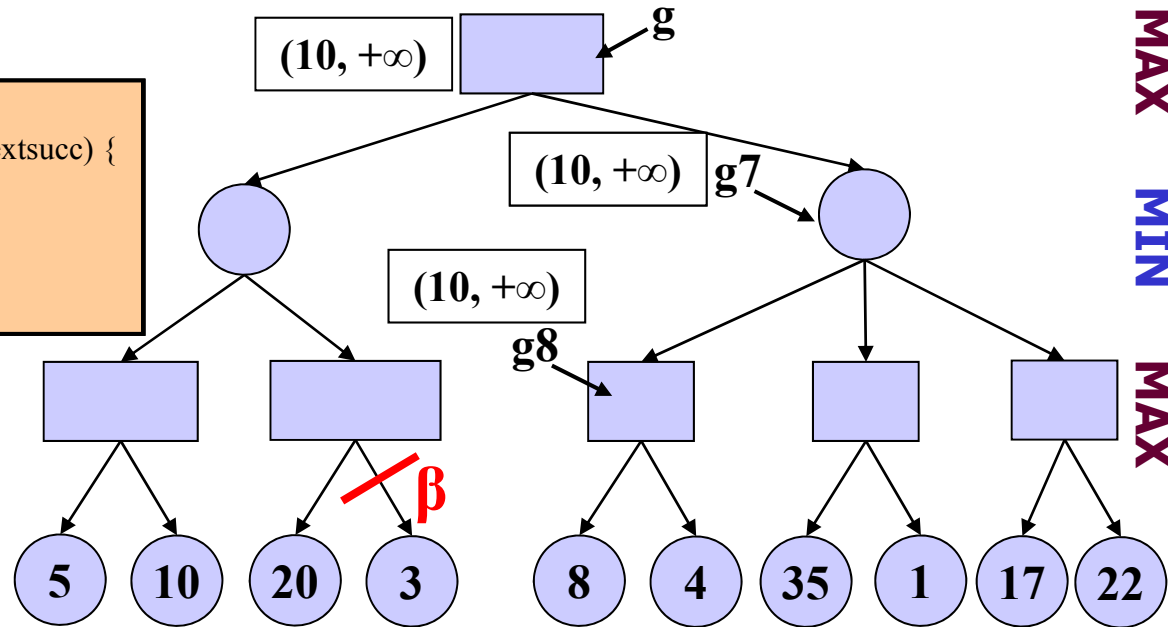
**maxValue(g8, 10, +∞)**

```
cutofftest(g8) → false  
for (GameState s = g8.firstsucc; s != g8.lastsucc; s = s.nextsucc) {  
    α = max(10, minValue(s, 10, +∞));  
    if (α ≥ +∞) break;  
}  
return α;
```

**g8 has no more successors!**

MAX  
MIN  
MAX

```
maxValue(g, -∞, +∞)  
cutofftest(g) → false  
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {  
    α = max(10, minValue(s, 10, +∞));  
    if (α ≥ +∞) break;  
}  
return α;
```



```
minValue(g7, 10, +∞)  
cutofftest(g7) → false  
for (GameState s = g7.firstsucc; s != g7.lastsucc; s = s.nextsucc) {  
    β = min(+∞, maxValue(s, 10, +∞));  
    if (β ≤ 10) break;  
}  
return β;
```

```
maxValue(g8, 10, +∞)  
cutofftest(g8) → false  
for (GameState s = g8.firstsucc; s != g8.lastsucc; s = s.nextsucc) {  
    α = 10;  
    if (10 ≥ +∞) break;  
}  
return 10;
```

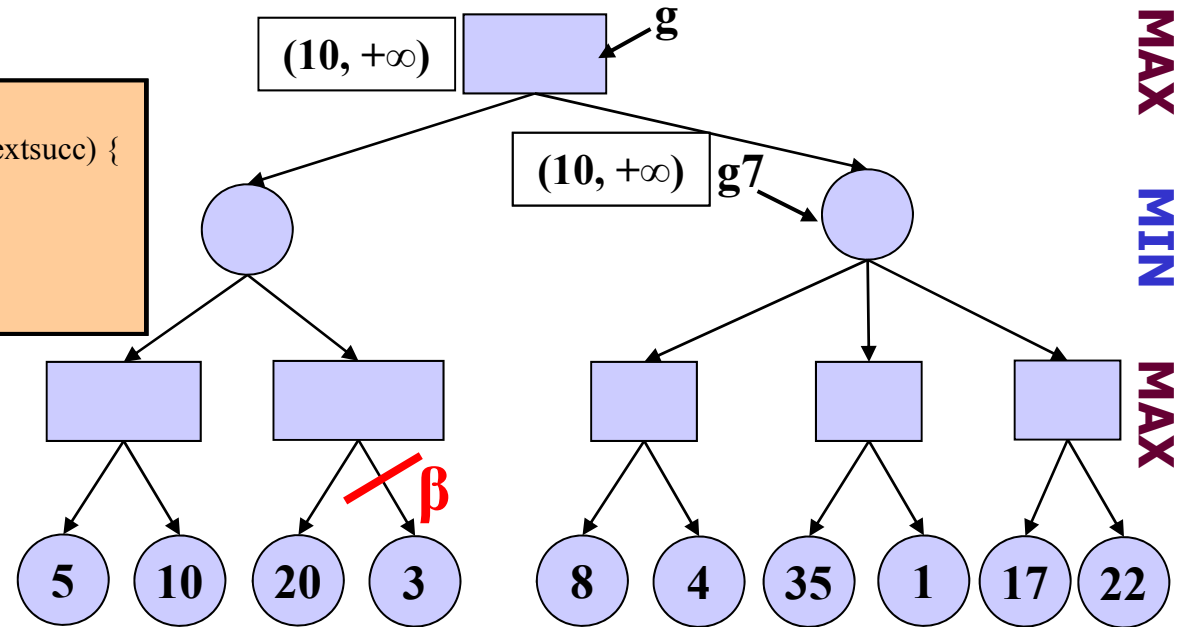
g8 has no more successors!

```
maxValue(g, -∞, +∞)
```

```
cutofftest(g) → false  
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {  
    α = max(10, minValue(s, 10, +∞));  
    if (α ≥ +∞) break;  
}  
return α;
```

```
minValue(g7, 10, +∞)
```

```
cutofftest(g7) → false  
for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {  
    β = min(+∞, 10);  
    if (β ≤ 10) break;  
}  
return β;
```



```

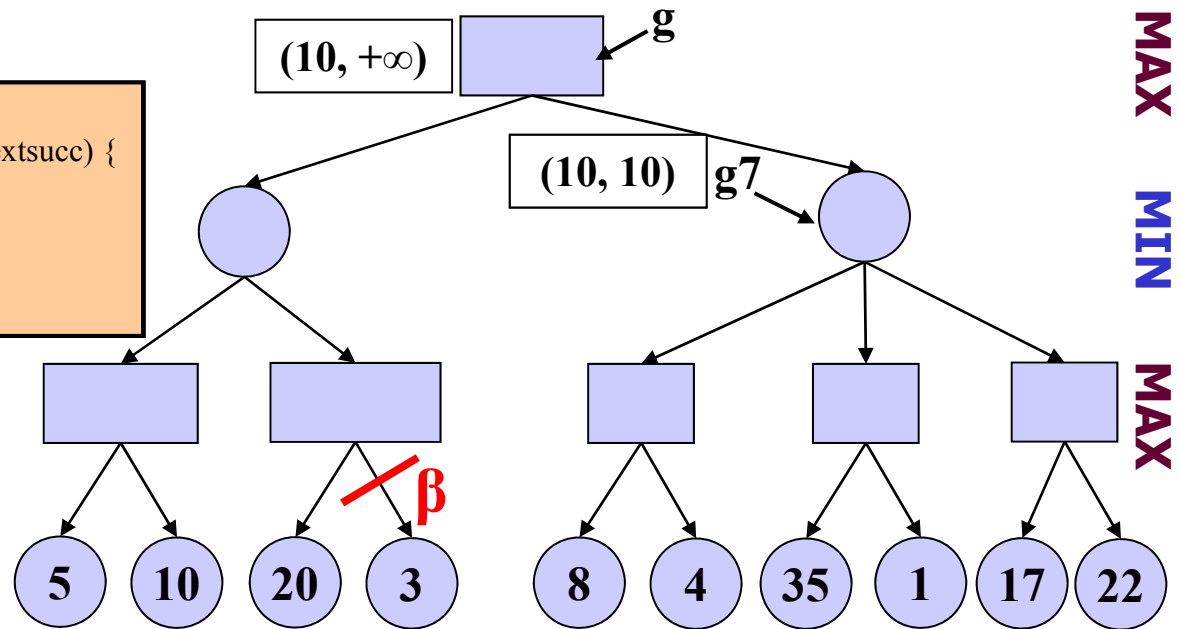
maxValue(g, -∞, +∞)
cutofftest(g) → false
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {
    α = max(10, minValue(s, 10, +∞));
    if ( α ≥ +∞ ) break;
}
return α;

```

```

minValue(g7, 10, +∞)
cutofftest(g7) → false
for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {
    β = 10;
    if ( β ≤ 10 ) break;
}
return β;

```

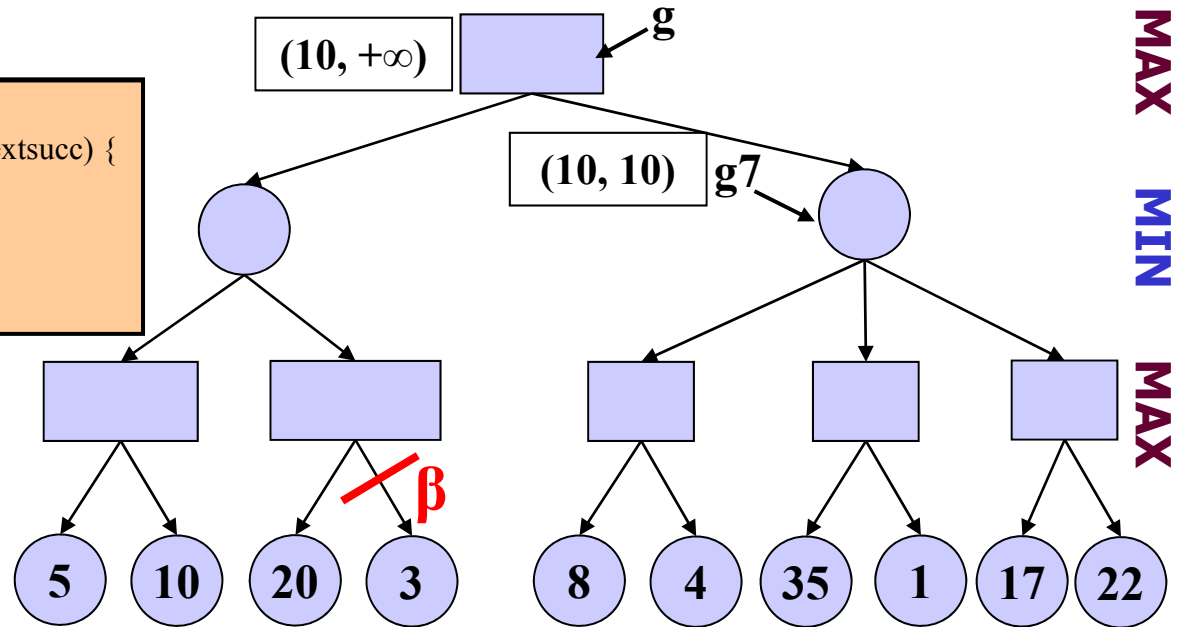


```
maxValue(g, -∞, +∞)
```

```
cutfftest(g) → false  
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {  
    α = max(10, minValue(s, 10, +∞));  
    if (α ≥ +∞) break;  
}  
return α;
```

```
minValue(g7, 10, +∞)
```

```
cutfftest(g7) → false  
for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {  
    β = 10;  
    if (10 ≤ 10) break;  
}  
return β; true!
```



```

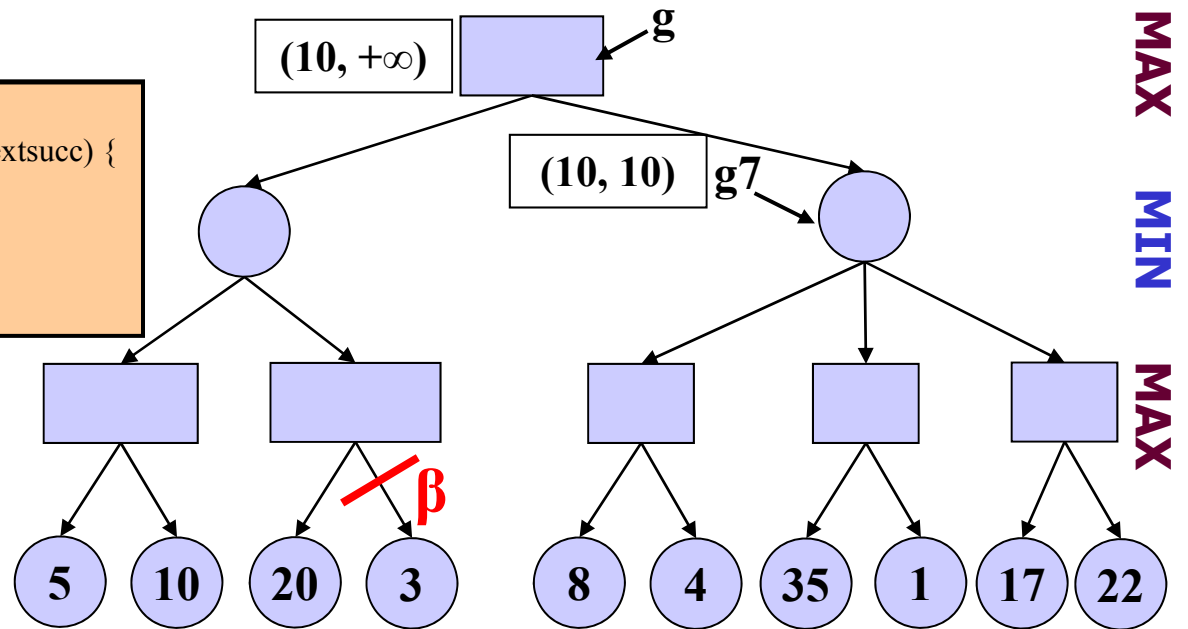
maxValue(g, -∞, +∞)
cutofftest(g) → false
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {
  α = max(10, minValue(s, 10, +∞));
  if (α ≥ +∞) break;
}
return α;

```

```

minValue(g7, 10, +∞)
cutofftest(g7) → false
for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {
  β = 10;
  if (10 ≤ 10) break;
}
return β;

```

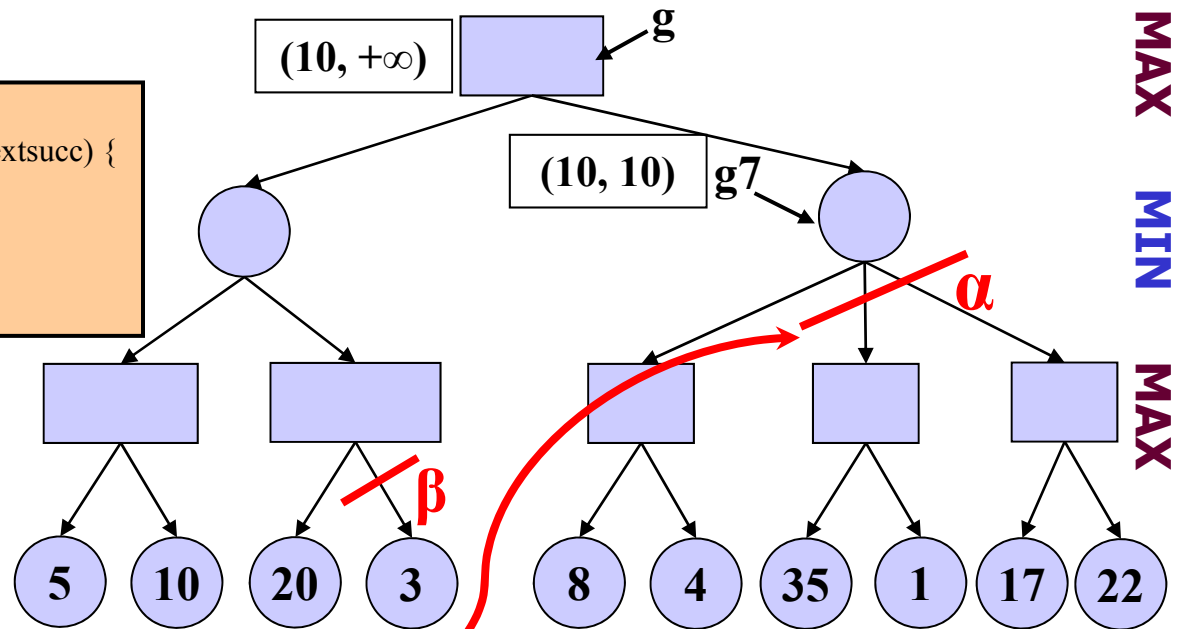


```
maxValue(g, -∞, +∞)
```

```
cutofftest(g) → false  
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {  
    α = max(10, minValue(s, 10, +∞));  
    if (α ≥ +∞) break;  
}  
return α;
```

```
minValue(g7, 10, +∞)
```

```
cutofftest(g7) → false  
for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {  
    β = 10;  
    if (10 ≤ 10) break; // α Schnitt!!  
}  
return 10;
```



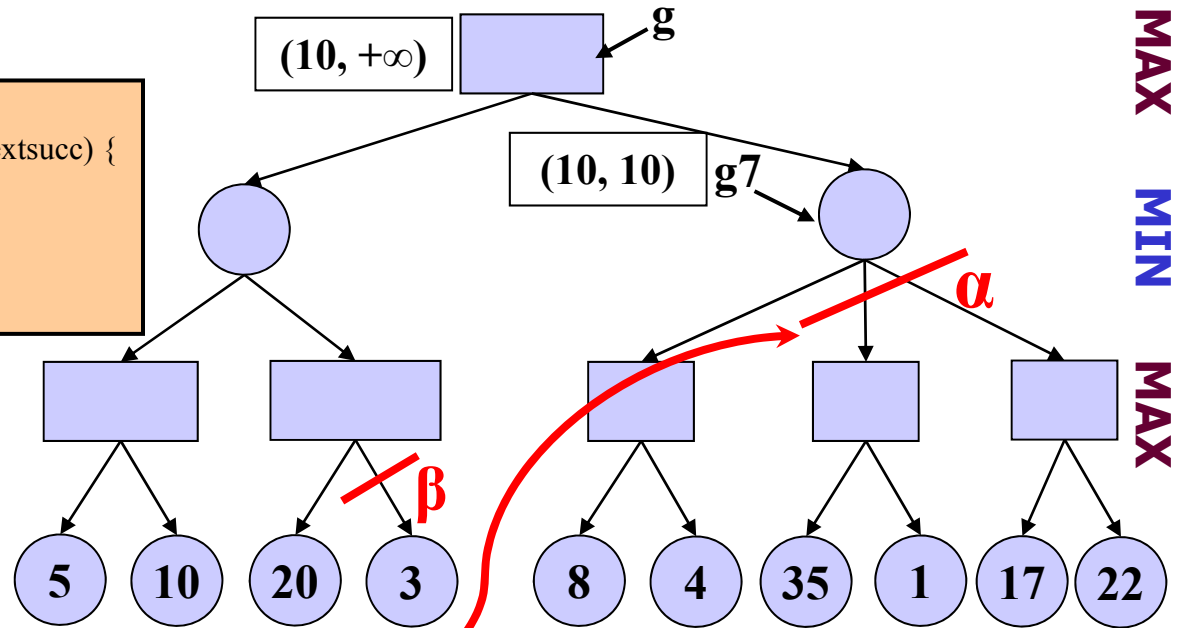
MAX  
MIN  
MAX

```
maxValue(g, -∞, +∞)
```

```
cutofftest(g) → false  
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {  
    α = max(10, minValue(s, 10, +∞));  
    if (α ≥ +∞) break;  
}  
return α;
```

```
minValue(g7, 10, +∞)
```

```
cutofftest(g7) → false  
for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {  
    β = 10;  
    if (10 ≤ 10) break; // α Schnitt!!  
}  
return 10;
```

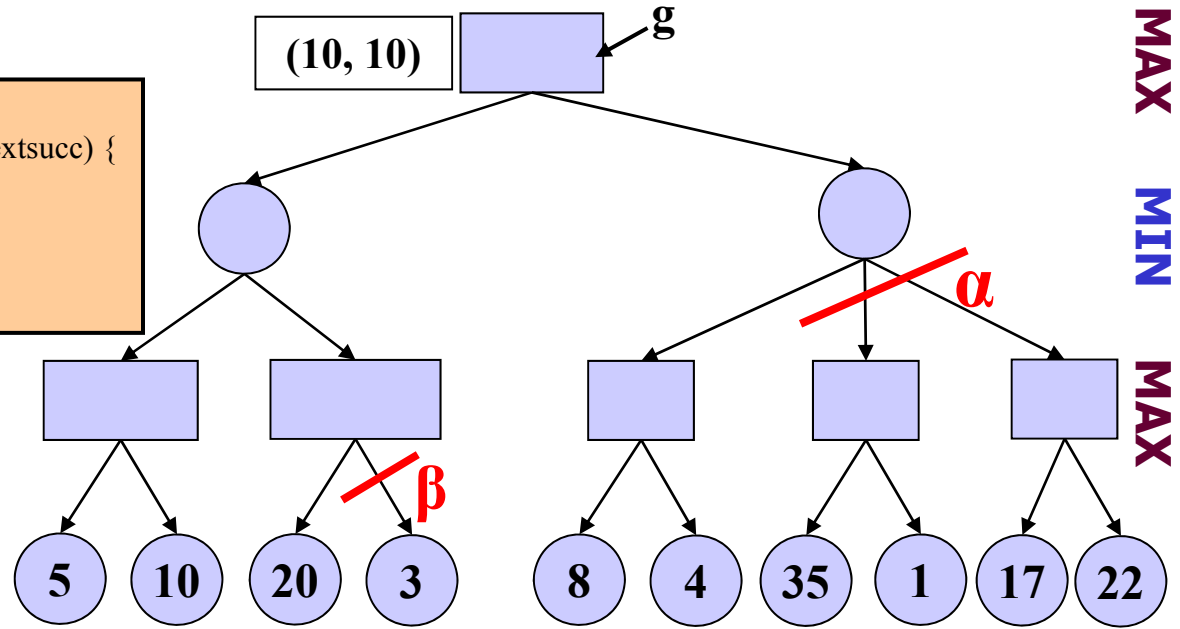






**maxValue(g, -∞, +∞)**

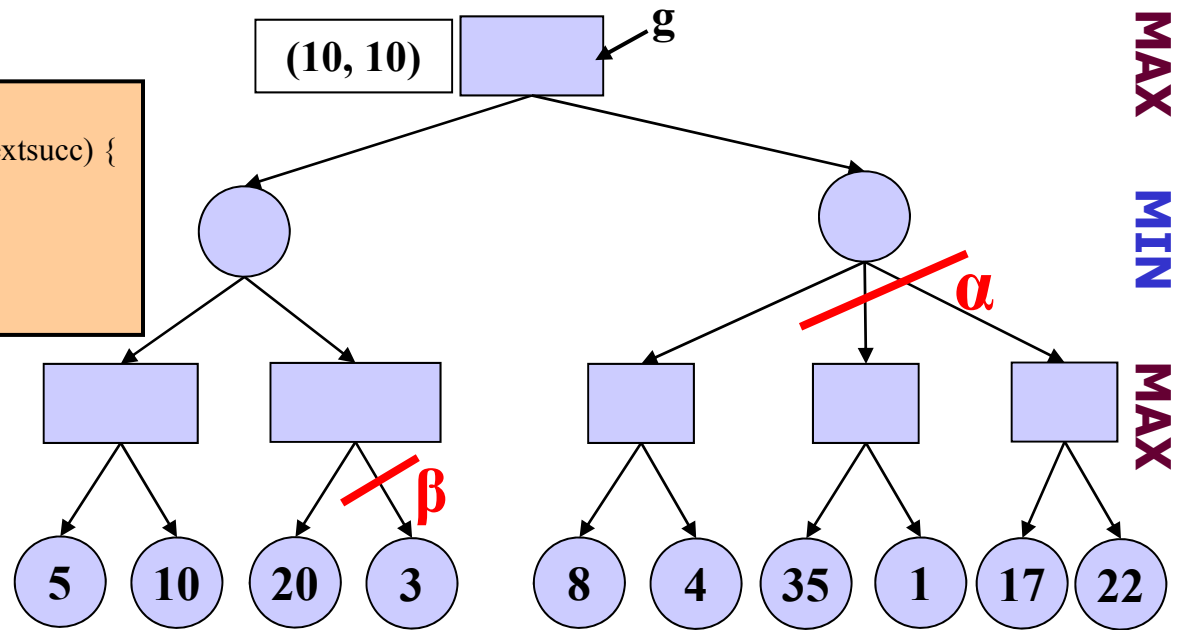
```
cutofftest(g) → false  
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {  
    α = 10  
    if (α ≥ +∞) break;  
}  
return α;
```



```

maxValue(g, -∞, +∞)
cutofftest(g) → false
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {
   $\alpha = 10;$ 
  if (  $10 \geq +\infty$  ) break;
}
return  $\alpha;$ 
false!

```

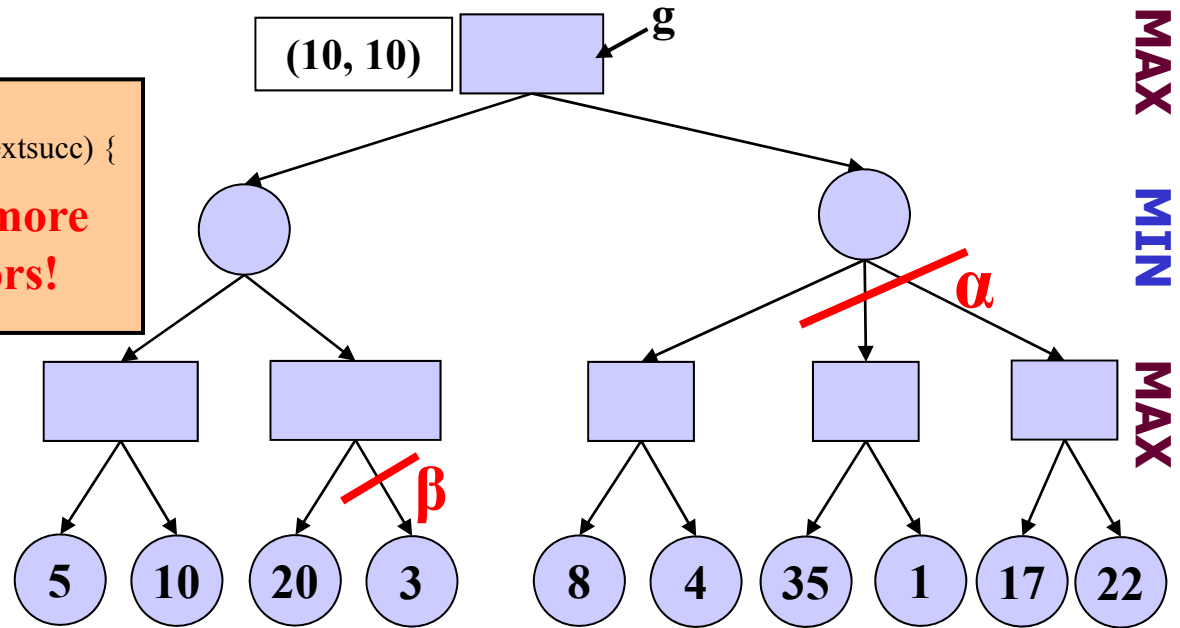


```

maxValue(g, -∞, +∞)
cutofftest(g) → false
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {
  α = 10;
  if (10 ≥ +∞) break;
}
return 10; ●

```

**g has no more successors!**



**maxValue(g, -∞, +∞)**

```
cutofftest(g) → false  
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {  
    α = 10;  
    if (10 ≥ +∞) break;  
}  
return 10;
```

**Maximum yield  
from current  
position is 10**

