

Übungsbeispiel α - β -Algorithmus

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Dr. Silvia Santini
Departement Informatik, ETH Zürich

Übungsbeispiel α - β -Algorithmus

- Illustriert auf 75 Folien Schritt für Schritt den Ablauf des α - β -Algorithmus

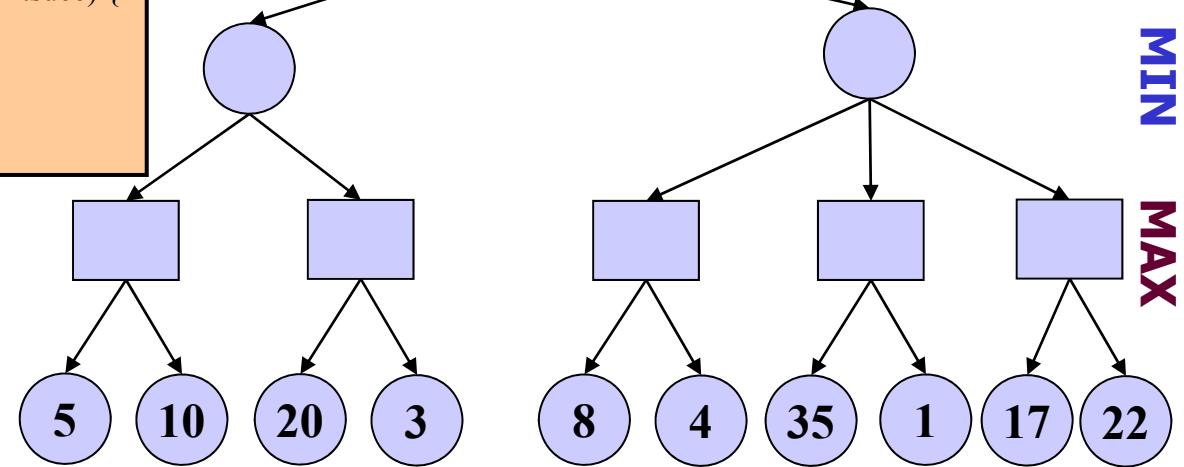
```
int maxValue(Gamestate g, int α, int β) {  
    if cutofftest(g) return eval(g);  
    for (GameState s=g.firstsucc; s!=g.lastsucc; s=s.nextsucc) {  
        α = max(α, minValue(s, α, β));  
        if (α >= β) break; // β-Schnitt  
    }  
    return α;  
}  
  
int minValue(Gamestate g, int α, int β) {  
    if cutofftest(g) return eval(g);  
    for (GameState s=g.firstsucc; s!=g.lastsucc; s=s.nextsucc) {  
        β = min(β, maxValue(s, α, β));  
        if (β <= α) break; // α-Schnitt  
    }  
    return β;  
}
```

```
maxValue(g, -∞, +∞)
```

```
cutofftest(g) → false  
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {  
    α = max(-∞, minValue(s, -∞, +∞));  
    if (α ≥ +∞) break;  
}  
return α;
```

(-∞, +∞)  \xrightarrow{g}

MAX
MIN
MAX



```
maxValue(g, -∞, +∞)
```

```
cutofftest(g) → false
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {
    α = max(-∞, minValue(s, -∞, +∞));
    if (α ≥ +∞) break;
}
return α;
```

```
(-∞, +∞)
```

```
g
```

```
g1
```

MAX

MIN

MAX

```
minValue(g1, -∞, +∞)
```

```
cutofftest(g1) → false
for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {
    β = min(+∞, maxValue(s, -∞, +∞));
    if (β ≤ -∞) break;
}
return β;
```

```
5
```

```
10
```

```
20
```

```
3
```

```
8
```

```
4
```

```
35
```

```
1
```

```
17
```

```
22
```

maxValue(g, -∞, +∞)

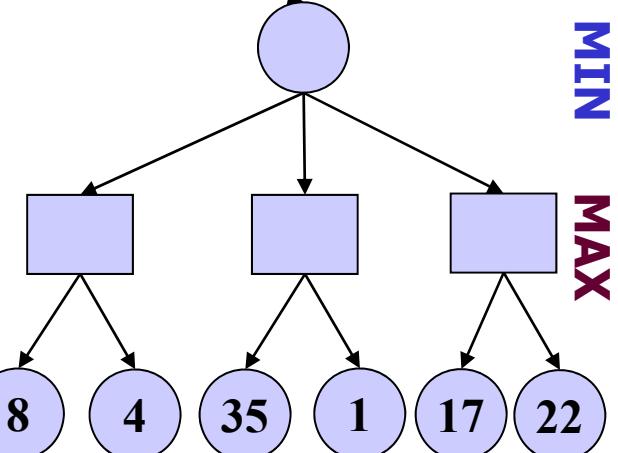
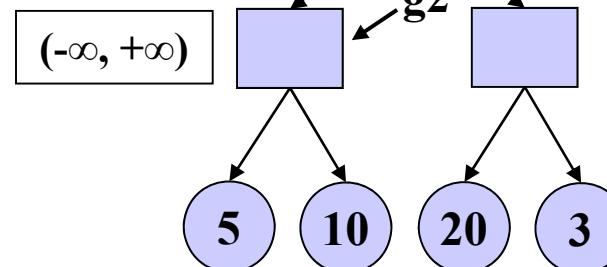
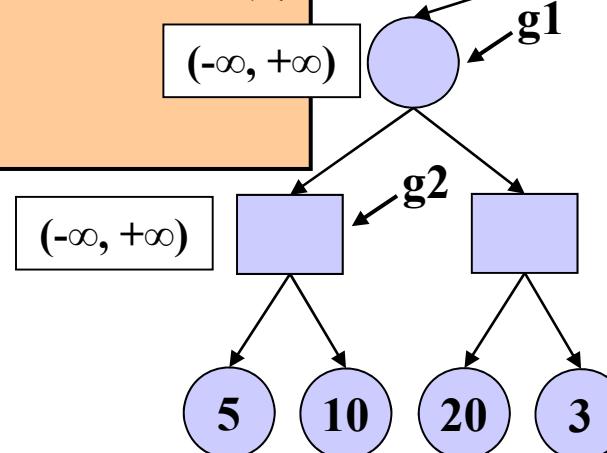
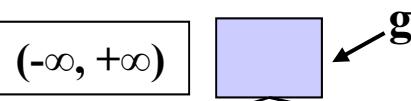
```
cutofftest(g) → false
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {
    α = max(-∞, minValue(s, -∞, +∞));
    if (α ≥ +∞) break;
}
return α;
```

minValue(g1, -∞, +∞)

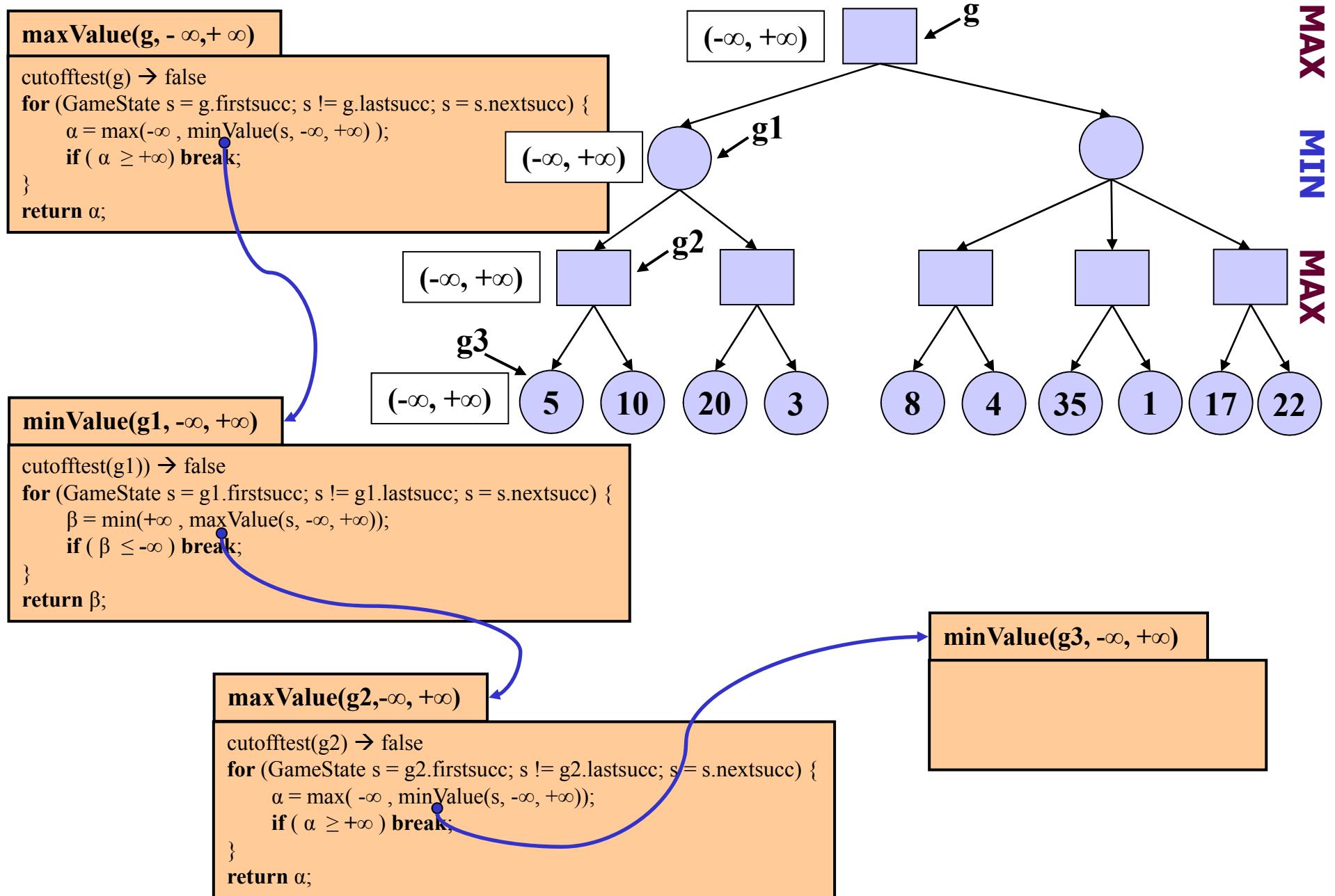
```
cutofftest(g1) → false
for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {
    β = min(+∞, maxValue(s, -∞, +∞));
    if (β ≤ -∞) break;
}
return β;
```

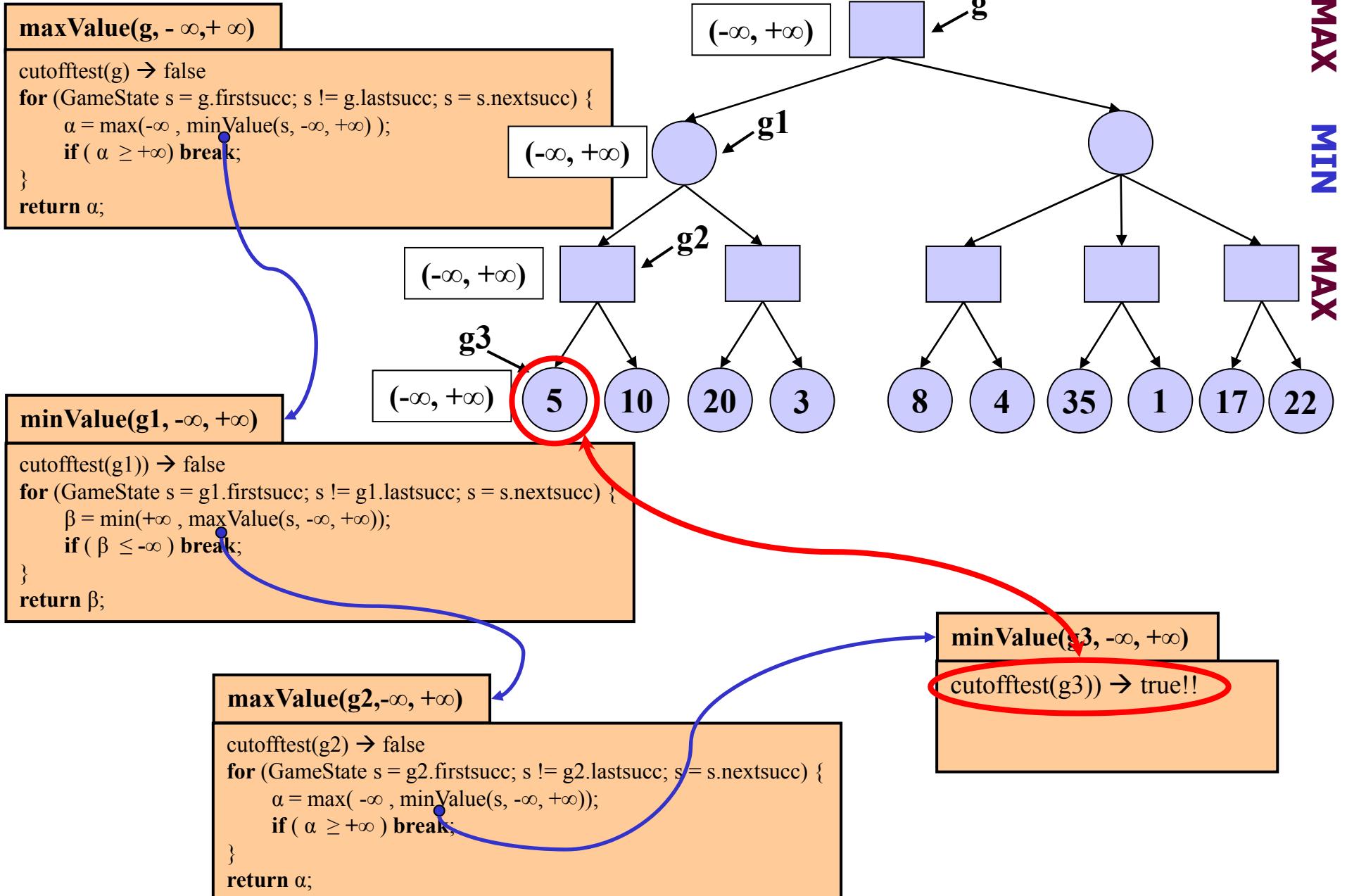
maxValue(g2, -∞, +∞)

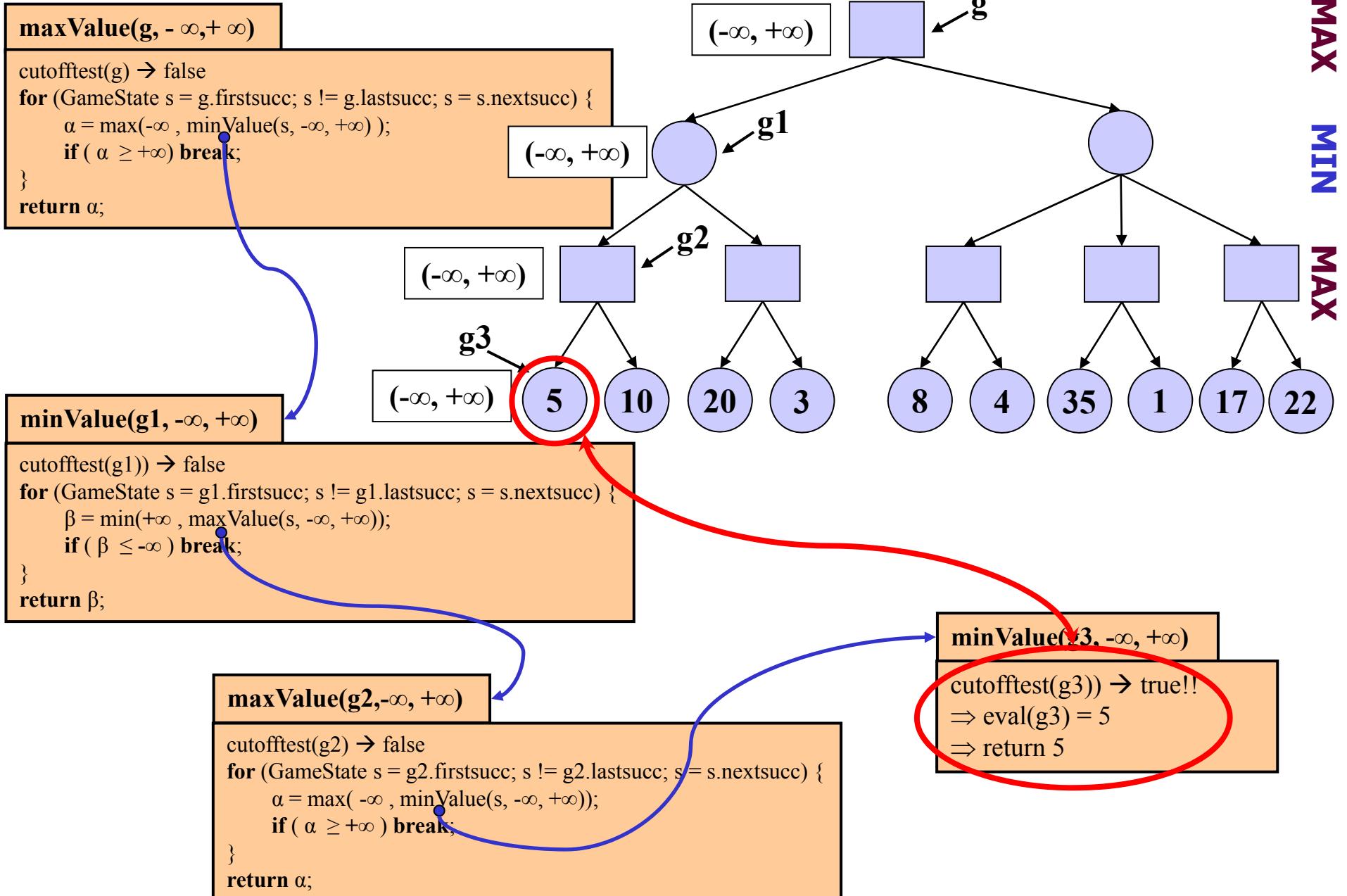
```
cutofftest(g2) → false
for (GameState s = g2.firstsucc; s != g2.lastsucc; s = s.nextsucc) {
    α = max(-∞, minValue(s, -∞, +∞));
    if (α ≥ +∞) break;
}
return α;
```



MAX
MIN
MAX







maxValue(g, -∞, +∞)

```
cutofftest(g) → false
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {
    α = max(-∞, minValue(s, -∞, +∞));
    if (α ≥ +∞) break;
}
return α;
```

minValue(g1, -∞, +∞)

```
cutofftest(g1)) → false
for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {
    β = min(+∞, maxValue(s, -∞, +∞));
    if (β ≤ -∞) break;
}
return β;
```

maxValue(g2,-∞, +∞)

```
cutofftest(g2) → false
for (GameState s = g2.firstsucc; s != g2.lastsucc; s = s.nextsucc) {
    α = max( -∞, minValue(s, -∞, +∞));
    if (α ≥ +∞) break;
}
return α;
```

(-∞, +∞)

(-∞, +∞)

g1

(-∞, +∞)

(-∞, +∞)

(-∞, +∞)

(-∞, +∞)

(-∞, +∞)

(-∞, +∞)

(-∞, +∞)

(-∞, +∞)

MAX

MIN

MAX

(-∞, +∞)

(-∞, +∞)

g2

(-∞, +∞)

(-∞, +∞)

(-∞, +∞)

(-∞, +∞)

(-∞, +∞)

(-∞, +∞)

(-∞, +∞)

(-∞, +∞)

g3

(-∞, +∞)

(-∞, +∞)

(-∞, +∞)

(-∞, +∞)

(-∞, +∞)

(-∞, +∞)

g

minValue(g3, -∞, +∞)

```
cutofftest(g3)) → true!!
⇒ eval(g3) = 5
⇒ return 5
```

maxValue(g, -∞, +∞)

```
cutofftest(g) → false
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {
    α = max(-∞, minValue(s, -∞, +∞));
    if (α ≥ +∞) break;
}
return α;
```

minValue(g1, -∞, +∞)

```
cutofftest(g1)) → false
for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {
    β = min(+∞, maxValue(s, -∞, +∞));
    if (β ≤ -∞) break;
}
return β;
```

maxValue(g2,-∞, +∞)

```
cutofftest(g2) → false
for (GameState s = g2.firstsucc; s != g2.lastsucc; s = s.nextsucc) {
    α = max(-∞, 5);
    if (α ≥ +∞) break;
}
return α;
```

(-∞, +∞)

(-∞, +∞)

(-∞, +∞)

(-∞, +∞)

5

10

20

3

8

4

35

1

17

22

MAX

MIN

MAX

minValue(g3, -∞, +∞)

```
cutofftest(g3)) → true!!
⇒ eval(g3) = 5
⇒ return 5
```

```
maxValue(g, -∞, +∞)
```

```
cutofftest(g) → false
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {
    α = max(-∞, minValue(s, -∞, +∞));
    if (α ≥ +∞) break;
}
return α;
```

```
(-∞, +∞)
```

```
(-∞, +∞)
```

g

MAX

MIN

MAX

```
minValue(g1, -∞, +∞)
```

```
cutofftest(g1)) → false
for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {
    β = min(+∞, maxValue(s, -∞, +∞));
    if (β ≤ -∞) break;
}
return β;
```

```
(-∞, +∞)
```

```
(-∞, +∞)
```

g1

5

10

20

3

```
(-∞, +∞)
```

```
(-∞, +∞)
```

g2

8

4

35

1

17

22

```
maxValue(g2, -∞, +∞)
```

```
cutofftest(g2) → false
for (GameState s = g2.firstsucc; s != g2.lastsucc; s = s.nextsucc) {
    α = max(-∞, 5);
    if (α ≥ +∞) break;
}
return α;
```

```
maxValue(g, -∞, +∞)
```

```
cutofftest(g) → false
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {
    α = max(-∞, minValue(s, -∞, +∞));
    if (α ≥ +∞) break;
}
return α;
```

(-∞, +∞)

g

MAX

```
minValue(g1, -∞, +∞)
```

```
cutofftest(g1)) → false
for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {
    β = min(+∞, maxValue(s, -∞, +∞));
    if (β ≤ -∞) break;
}
return β;
```

(-∞, +∞)

(5, +∞)

g1

5 10

g2

20 3

MIN

MAX

8 4

35 1

17 22

```
maxValue(g2,-∞, +∞)
```

```
cutofftest(g2) → false
for (GameState s = g2.firstsucc; s != g2.lastsucc; s = s.nextsucc) {
    a = 5;
    if (a ≥ +∞) break;
}
return a;
```

MIN

maxValue(g, -∞, +∞)

```
cutofftest(g) → false
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {
    α = max(-∞, minValue(s, -∞, +∞));
    if (α ≥ +∞) break;
}
return α;
```

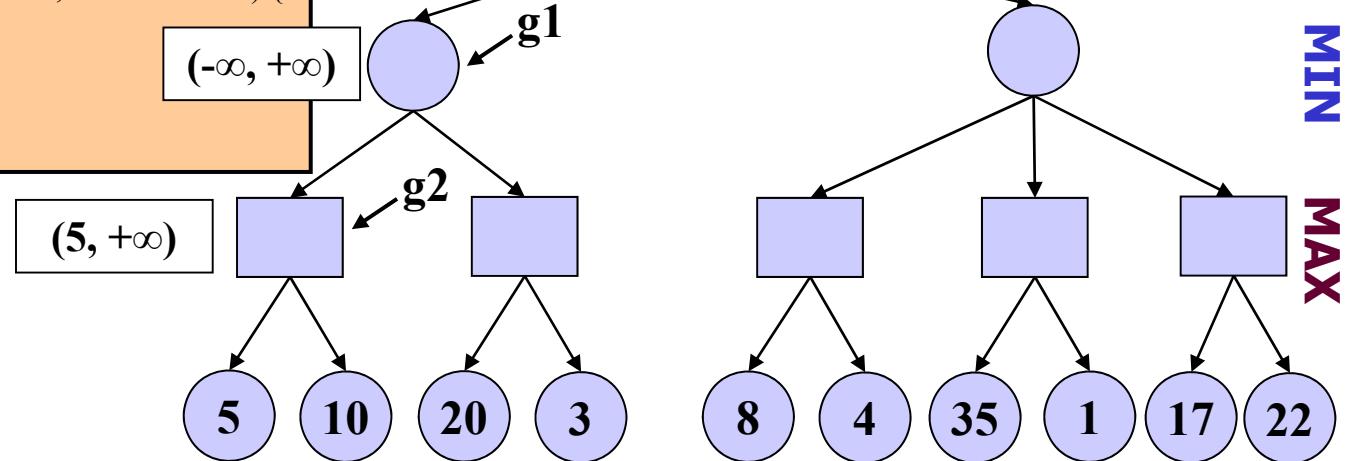
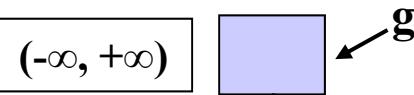
minValue(g1, -∞, +∞)

```
cutofftest(g1) → false
for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {
    β = min(+∞, maxValue(s, -∞, +∞));
    if (β ≤ -∞) break;
}
return β;
```

maxValue(g2, -∞, +∞)

```
cutofftest(g2) → false
for (GameState s = g2.firstsucc; s != g2.lastsucc; s = s.nextsucc) {
    g = 5;
    α = max(-∞, minValue(s, -∞, +∞));
    if (α ≥ +∞) break;
}
return α;
```

false!



MAX
MIN
MAX

maxValue(g , $-\infty$, $+\infty$)

```
cutofftest( $g$ ) → false
for (GameState s =  $g$ .firstsucc; s !=  $g$ .lastsucc; s = s.nextsucc) {
     $\alpha$  = max( $-\infty$ , minValue(s,  $-\infty$ ,  $+\infty$ ));
    if ( $\alpha \geq +\infty$ ) break;
}
return  $\alpha$ ;
```

$(-\infty, +\infty)$

$(-\infty, +\infty)$

g

MAX

MIN

MAX

minValue(g_1 , $-\infty$, $+\infty$)

```
cutofftest( $g_1$ ) → false
for (GameState s =  $g_1$ .firstsucc; s !=  $g_1$ .lastsucc; s = s.nextsucc) {
     $\beta$  = min( $+\infty$ , maxValue(s,  $-\infty$ ,  $+\infty$ ));
    if ( $\beta \leq -\infty$ ) break;
}
return  $\beta$ ;
```

$(5, +\infty)$

$(-\infty, +\infty)$

$(-\infty, +\infty)$

$(-\infty, +\infty)$

5

10

20

3

$(-\infty, +\infty)$

$(-\infty, +\infty)$

$(-\infty, +\infty)$

8

4

35

1

17

22

maxValue(g_2 , $-\infty$, $+\infty$)

```
cutofftest( $g_2$ ) → false
for (GameState s =  $g_2$ .firstsucc; s !=  $g_2$ .lastsucc; s = s.nextsucc) {
     $\alpha$  = max( 5, minValue(s, 5,  $+\infty$ ));
    if ( $\alpha \geq +\infty$ ) break;
}
return  $\alpha$ ;
```

next successor

maxValue(g, -∞, +∞)

```
cutofftest(g) → false
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {
    α = max(-∞, minValue(s, -∞, +∞));
    if (α ≥ +∞) break;
}
return α;
```

minValue(g1, -∞, +∞)

```
cutofftest(g1)) → false
for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {
    β = min(+∞, maxValue(s, -∞, +∞));
    if (β ≤ -∞) break;
}
return β;
```

maxValue(g2,-∞, +∞)

```
cutofftest(g2) → false
for (GameState s = g2.firstsucc; s != g2.lastsucc; s = s.nextsucc) {
    α = max( 5 , minValue(s, 5 , +∞));
    if (α ≥ +∞) break;
}
return α;
```

(-∞, +∞)

(-∞, +∞)

(-∞, +∞)

(-∞, +∞)

(5, +∞)

5

10

20

3

(-∞, +∞)

8

4

35

1

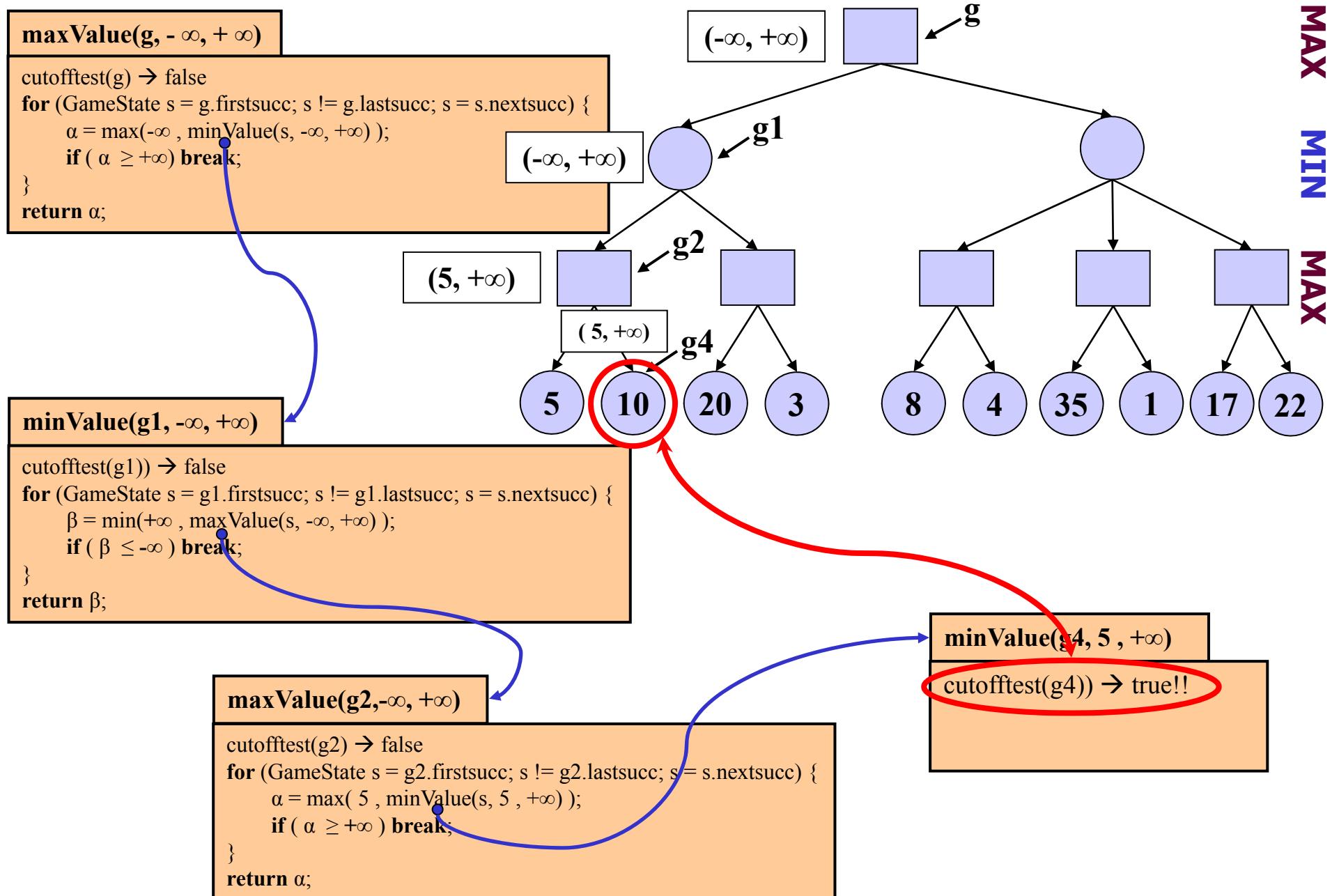
17

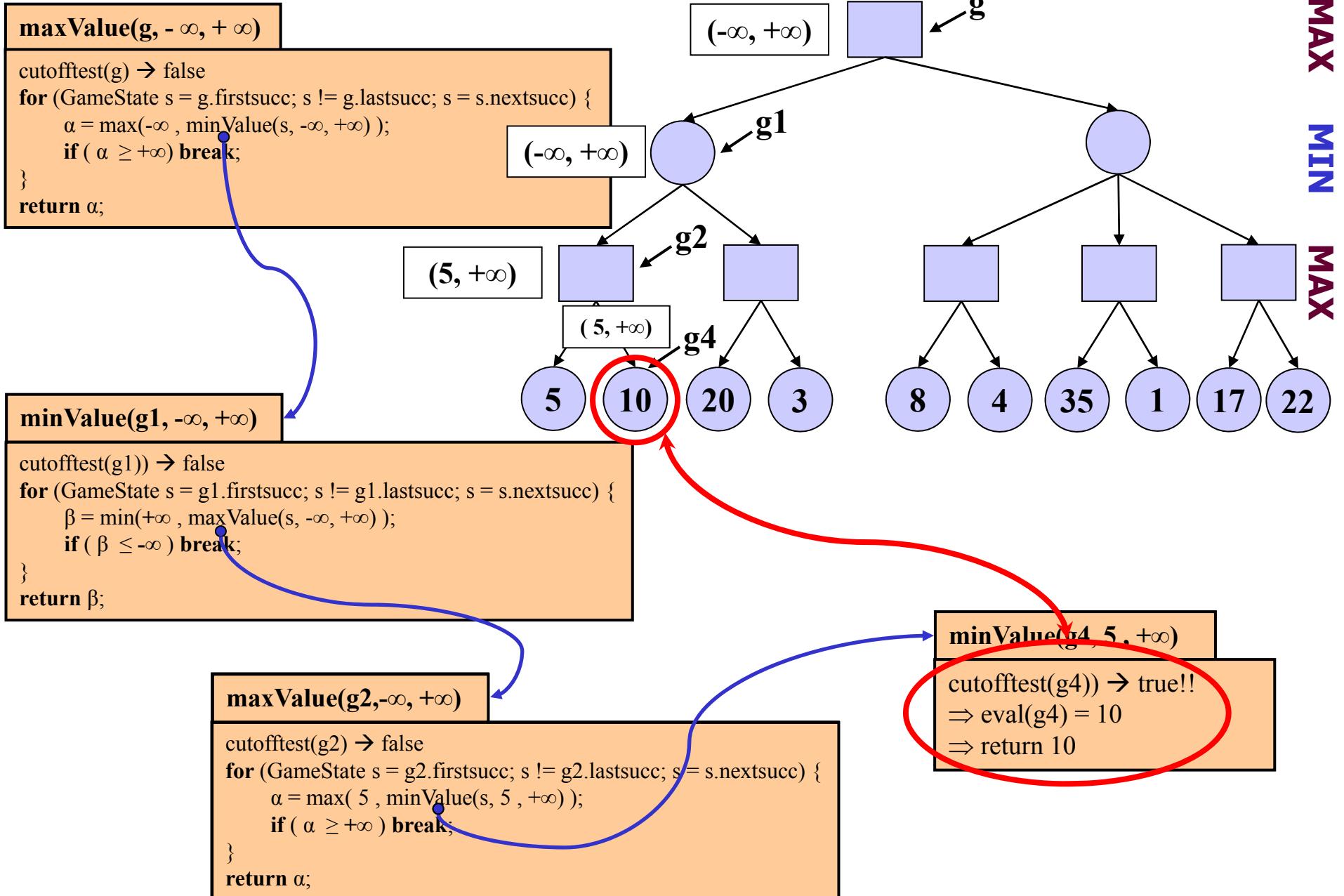
22

MAX

MIN

MAX





maxValue(g, -∞, +∞)

```
cutofftest(g) → false
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {
    α = max(-∞, minValue(s, -∞, +∞));
    if (α ≥ +∞) break;
}
return α;
```

minValue(g1, -∞, +∞)

```
cutofftest(g1)) → false
for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {
    β = min(+∞, maxValue(s, -∞, +∞));
    if (β ≤ -∞) break;
}
return β;
```

maxValue(g2,-∞, +∞)

```
cutofftest(g2) → false
for (GameState s = g2.firstsucc; s != g2.lastsucc; s = s.nextsucc) {
    α = max(5, minValue(s, 5, +∞));
    if (α ≥ +∞) break;
}
return α;
```

(-∞, +∞)

(-∞, +∞)

(-∞, +∞)

(5, +∞)

5

10

20

3

(-∞, +∞)

8

4

35

1

17

22

(5, +∞)

g1

(5, +∞)

g2

g4

MAX

MIN

MAX

minValue(g4, 5 , +∞)

```
cutofftest(g4)) → true!!
⇒ eval(g4) = 10
⇒ return 10.
```

maxValue(g , $-\infty$, $+\infty$)

```
cutofftest( $g$ ) → false
for (GameState s =  $g$ .firstsucc; s !=  $g$ .lastsucc; s = s.nextsucc) {
     $\alpha$  = max( $-\infty$ , minValue(s,  $-\infty$ ,  $+\infty$ ));
    if ( $\alpha \geq +\infty$ ) break;
}
return  $\alpha$ ;
```

minValue(g_1 , $-\infty$, $+\infty$)

```
cutofftest( $g_1$ ) → false
for (GameState s =  $g_1$ .firstsucc; s !=  $g_1$ .lastsucc; s = s.nextsucc) {
     $\beta$  = min( $+\infty$ , maxValue(s,  $-\infty$ ,  $+\infty$ ));
    if ( $\beta \leq -\infty$ ) break;
}
return  $\beta$ ;
```

maxValue(g_2 , $-\infty$, $+\infty$)

```
cutofftest( $g_2$ ) → false
for (GameState s =  $g_2$ .firstsucc; s !=  $g_2$ .lastsucc; s = s.nextsucc) {
     $\alpha$  = max(5, 10);
    if ( $\alpha \geq +\infty$ ) break;
}
return  $\alpha$ ;
```

$(-\infty, +\infty)$

$(-\infty, +\infty)$

$(-\infty, +\infty)$

$(5, +\infty)$

5

10

20

3

$(-\infty, +\infty)$

8

4

35

1

17

22

MAX
MIN
MAX

g

g_1

g_2

g_4

minValue(g_4 , 5, $+\infty$)

```
cutofftest( $g_4$ ) → true!!
⇒ eval( $g_4$ ) = 10
⇒ return 10
```

maxValue(g, -∞, +∞)

```
cutofftest(g) → false
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {
    α = max(-∞, minValue(s, -∞, +∞));
    if (α ≥ +∞) break;
}
return α;
```

(-∞, +∞)

g

(-∞, +∞)

(10, +∞)

5 10

g1

g2

20 3

MAX
MIN
MAX

8 4

35 1

g

17 22

minValue(g1, -∞, +∞)

```
cutofftest(g1) → false
for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {
    β = min(+∞, maxValue(s, -∞, +∞));
    if (β ≤ -∞) break;
}
return β;
```

maxValue(g2, -∞, +∞)

```
cutofftest(g2) → false
for (GameState s = g2.firstsucc; s != g2.lastsucc; s = s.nextsucc) {
    α = 10;
    if (α ≥ +∞) break;
}
return α;
```

maxValue(g, -∞, +∞)

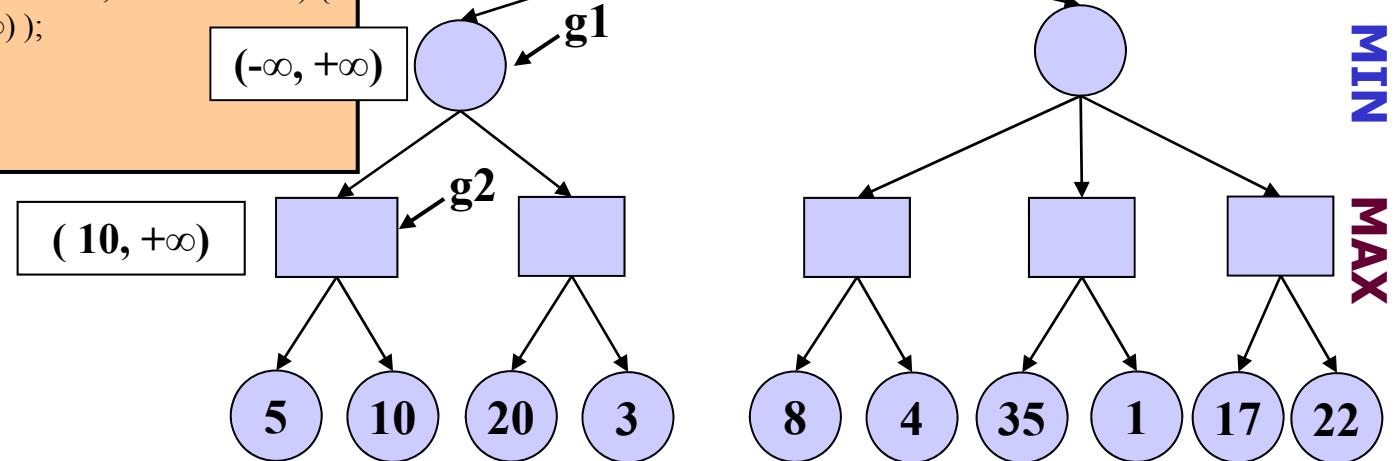
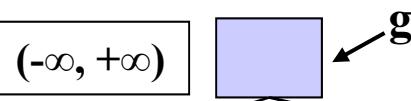
```
cutofftest(g) → false
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {
    α = max(-∞, minValue(s, -∞, +∞));
    if (α ≥ +∞) break;
}
return α;
```

minValue(g1, -∞, +∞)

```
cutofftest(g1) → false
for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {
    β = min(+∞, maxValue(s, -∞, +∞));
    if (β ≤ -∞) break;
}
return β;
```

maxValue(g2, -∞, +∞)

```
cutofftest(g2) → false
for (GameState s = g2.firstsucc; s != g2.lastsucc; s = s.nextsucc) {
    α = max(-∞, minValue(s, -∞, +∞));
    if (α ≥ +∞) break;
}
return α;
```



maxValue(g, -∞, +∞)

```
cutofftest(g) → false
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {
    α = max(-∞, minValue(s, -∞, +∞));
    if (α ≥ +∞) break;
}
return α;
```

minValue(g1, -∞, +∞)

```
cutofftest(g1) → false
for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {
    β = min(+∞, maxValue(s, -∞, +∞));
    if (β ≤ -∞) break;
}
return β;
```

maxValue(g2, -∞, +∞)

```
cutofftest(g2) → false
for (GameState s = g2.firstsucc; s != g2.lastsucc; s = s.nextsucc) {
    α = 10;
    if (10 ≥ +∞) break;
}
return α;
```

(-∞, +∞)

g1

(10, +∞)

g2

MAX
MIN
MAX

5

10

20

3

8

4

35

1

17

22

g2 has no more successors!

maxValue(g, -∞, +∞)

```
cutofftest(g) → false
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {
    α = max(-∞, minValue(s, -∞, +∞));
    if (α ≥ +∞) break;
}
return α;
```

minValue(g1, -∞, +∞)

```
cutofftest(g1) → false
for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {
    β = min(+∞, maxValue(s, -∞, +∞));
    if (β ≤ -∞) break;
}
return β;
```

maxValue(g2, -∞, +∞)

```
cutofftest(g2) → false
for (GameState s = g2.firstsucc; s != g2.lastsucc; s = s.nextsucc) {
    α = 10;
    if (10 ≥ +∞) break;
}
return α;
```

(-∞, +∞)

g1

(10, +∞)

g2

MAX
MIN
MAX

5

10

20

3

8

4

35

1

17

22

g2 has no more successors!

maxValue(g, -∞, +∞)

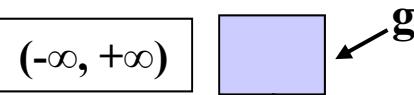
```
cutofftest(g) → false
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {
    α = max(-∞, minValue(s, -∞, +∞));
    if (α ≥ +∞) break;
}
return α;
```

minValue(g1, -∞, +∞)

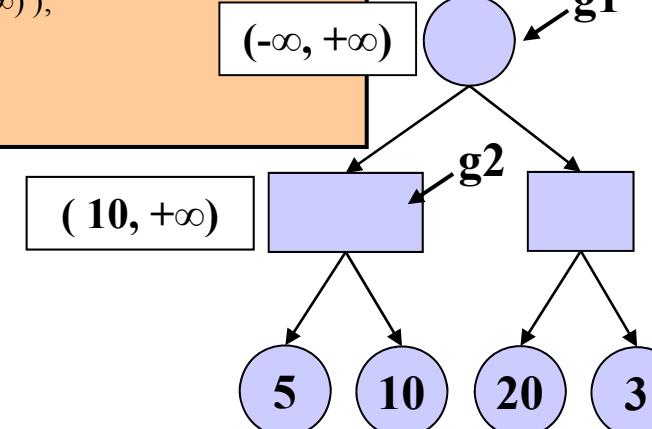
```
cutofftest(g1) → false
for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {
    β = min(+∞, maxValue(s, -∞, +∞));
    if (β ≤ -∞) break;
}
return β;
```

maxValue(g2, -∞, +∞)

```
cutofftest(g2) → false
for (GameState s = g2.firstsucc; s != g2.lastsucc; s = s.nextsucc) {
    α = 10;
    if (10 ≥ +∞) break;
}
return 10;
```



g1



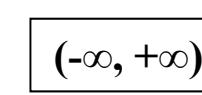
(10, +∞)

5

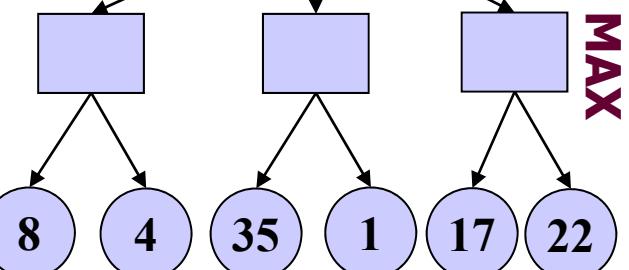
10

20

3



g



8

4

35

1

17

22

MAX
MIN
MAX

maxValue(g, -∞, +∞)

```
cutofftest(g) → false
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {
    α = max(-∞, minValue(s, -∞, +∞));
    if (α ≥ +∞) break;
}
return α;
```

minValue(g1, -∞, +∞)

```
cutofftest(g1) → false
for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {
    β = min(+∞, maxValue(s, -∞, +∞));
    if (β ≤ -∞) break;
}
return β;
```

maxValue(g2, -∞, +∞)

```
cutofftest(g2) → false
for (GameState s = g2.firstsucc; s != g2.lastsucc; s = s.nextsucc) {
    α = 10;
    if (10 ≥ +∞) break;
}
return 10;
```

(-∞, +∞)

g1

(-∞, +∞)

(10, +∞)

g2

5

10

20

3

8

4

35

1

17

22

MAX

MIN

MAX

g2 has no more successors!

maxValue(g, -∞, +∞)

```
cutofftest(g) → false
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {
    α = max(-∞, minValue(s, -∞, +∞));
    if (α ≥ +∞) break;
}
return α;
```

minValue(g1, -∞, +∞)

```
cutofftest(g1) → false
for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {
    β = min(+∞, 10);
    if (β ≤ -∞) break;
}
return β;
```

maxValue(g2, -∞, +∞)

```
cutofftest(g2) → false
for (GameState s = g2.firstsucc; s != g2.lastsucc; s = s.nextsucc) {
    α = 10;
    if (10 ≥ +∞) break;
}
return 10;
```

(-∞, +∞)

(-∞, +∞)

(-∞, +∞)

(10, +∞)

5 10

20 3

8 4

35 1

17 22

MAX

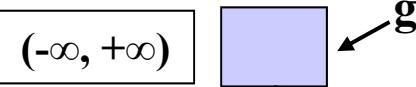
MIN

MAX

```
maxValue(g, -∞, +∞)
```

```
cutofftest(g) → false  
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {  
    α = max(-∞, minValue(s, -∞, +∞));  
    if (α ≥ +∞) break;  
}  
return α;
```

```
(-∞, +∞)
```



MAX

MIN

MAX

```
minValue(g1, -∞, +∞)
```

```
cutofftest(g1) → false  
for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {  
    β = min(+∞, 10);  
    if (β ≤ -∞) break;  
}  
return β;
```

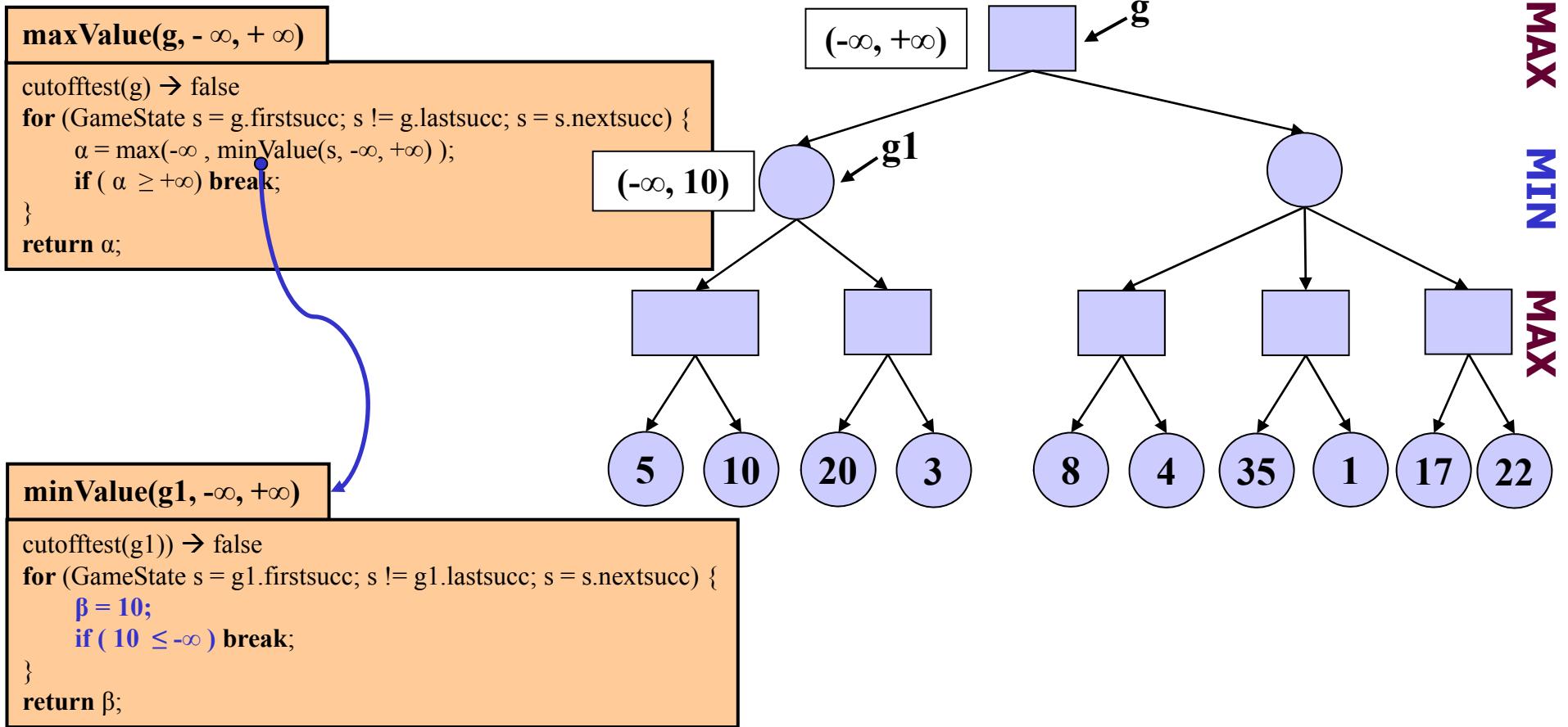
```
5 10
```

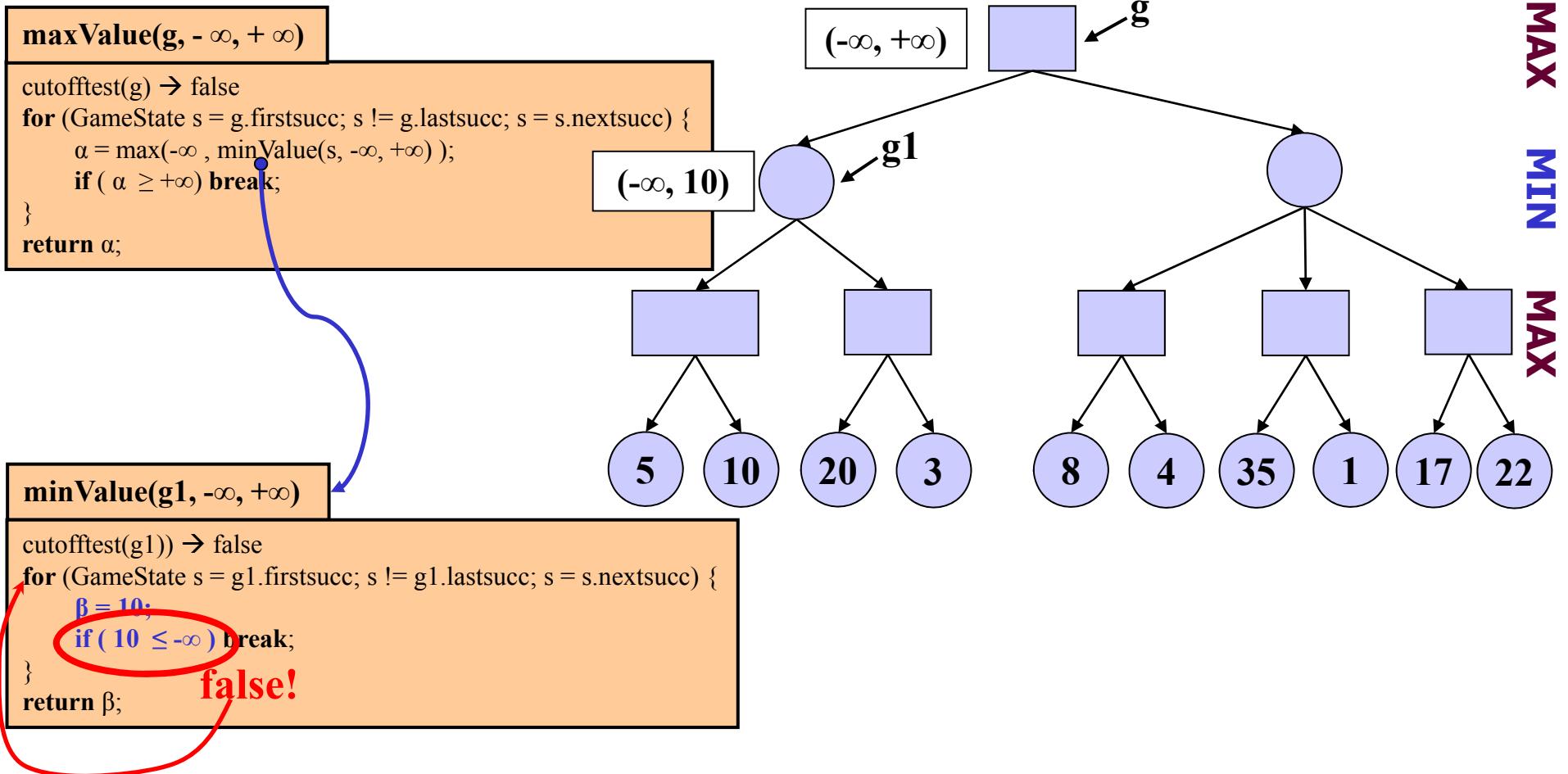
```
20 3
```

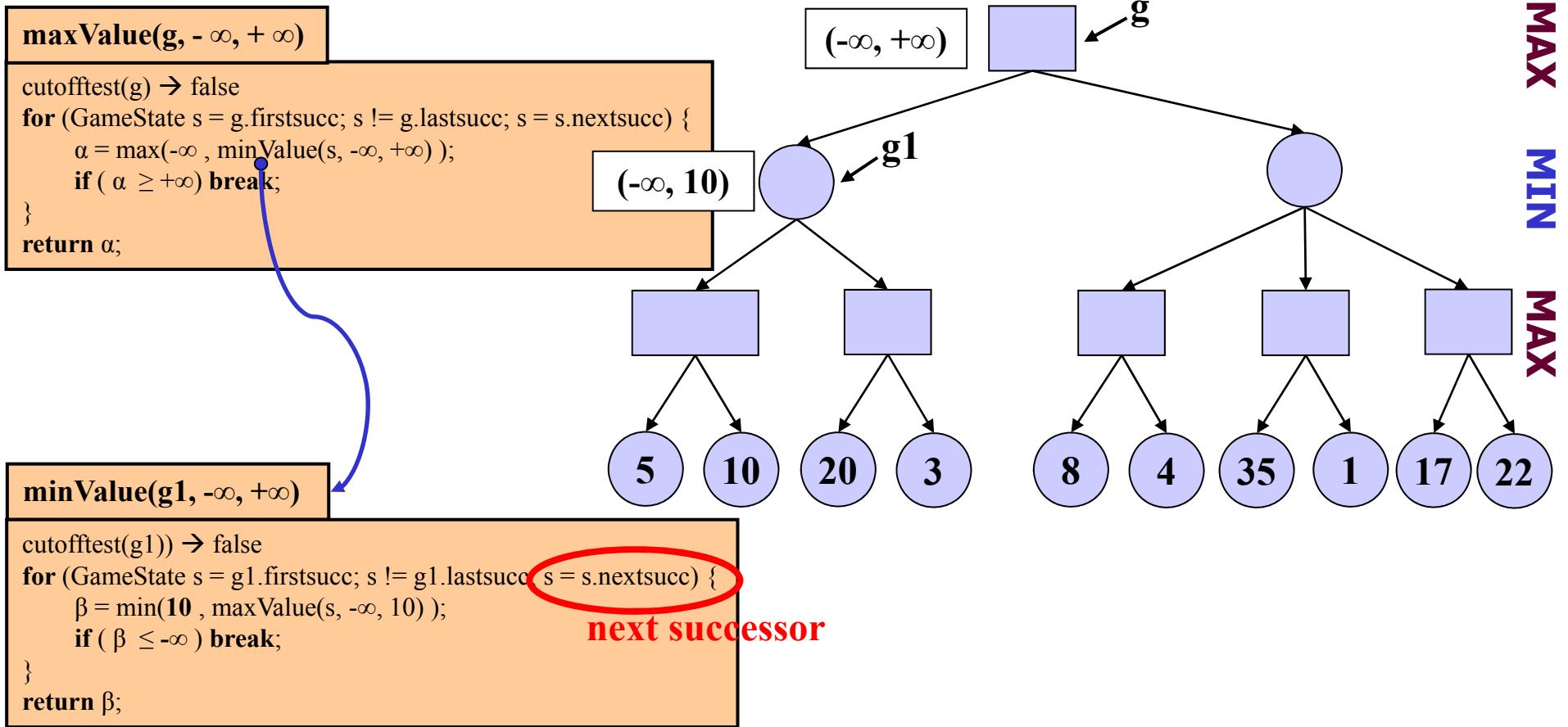
```
8 4
```

```
35 1
```

```
17 22
```







maxValue(g, -∞, +∞)

```
cutofftest(g) → false
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {
    α = max(-∞, minValue(s, -∞, +∞));
    if (α ≥ +∞) break;
}
return α;
```

minValue(g1, -∞, +∞)

```
cutofftest(g1) → false
for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {
    β = min(10, maxValue(s, -∞, 10));
    if (β ≤ -∞) break;
}
return β;
```

maxValue(g5, -∞, 10)

```
cutofftest(g5) → false
for (GameState s = g5.firstsucc; s != g5.lastsucc; s = s.nextsucc) {
    α = max(-∞, minValue(s, -∞, 10));
    if (α ≥ 10) break;
}
return α;
```

(-∞, +∞)

g

(-∞, 10)

(-∞, 10)

5 10

20 3

8 4

35 1

17 22

g1

g5

MAX
MIN
MAX

maxValue(g, -∞, +∞)

```
cutofftest(g) → false
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {
    α = max(-∞, minValue(s, -∞, +∞));
    if (α ≥ +∞) break;
}
return α;
```

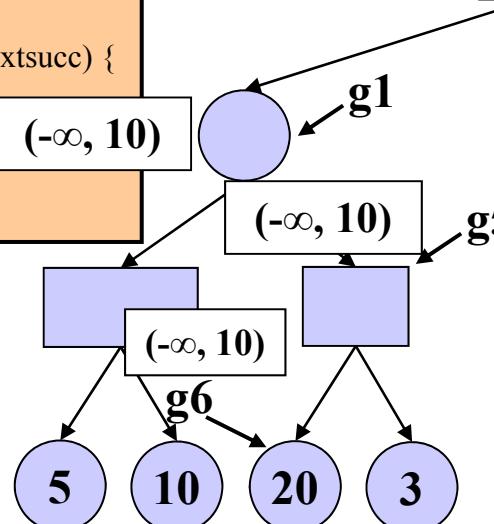
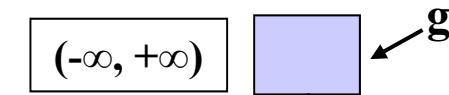
minValue(g1, -∞, +∞)

```
cutofftest(g1)) → false
for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {
    β = min(10, maxValue(s, -∞, 10));
    if (β ≤ -∞) break;
}
return β;
```

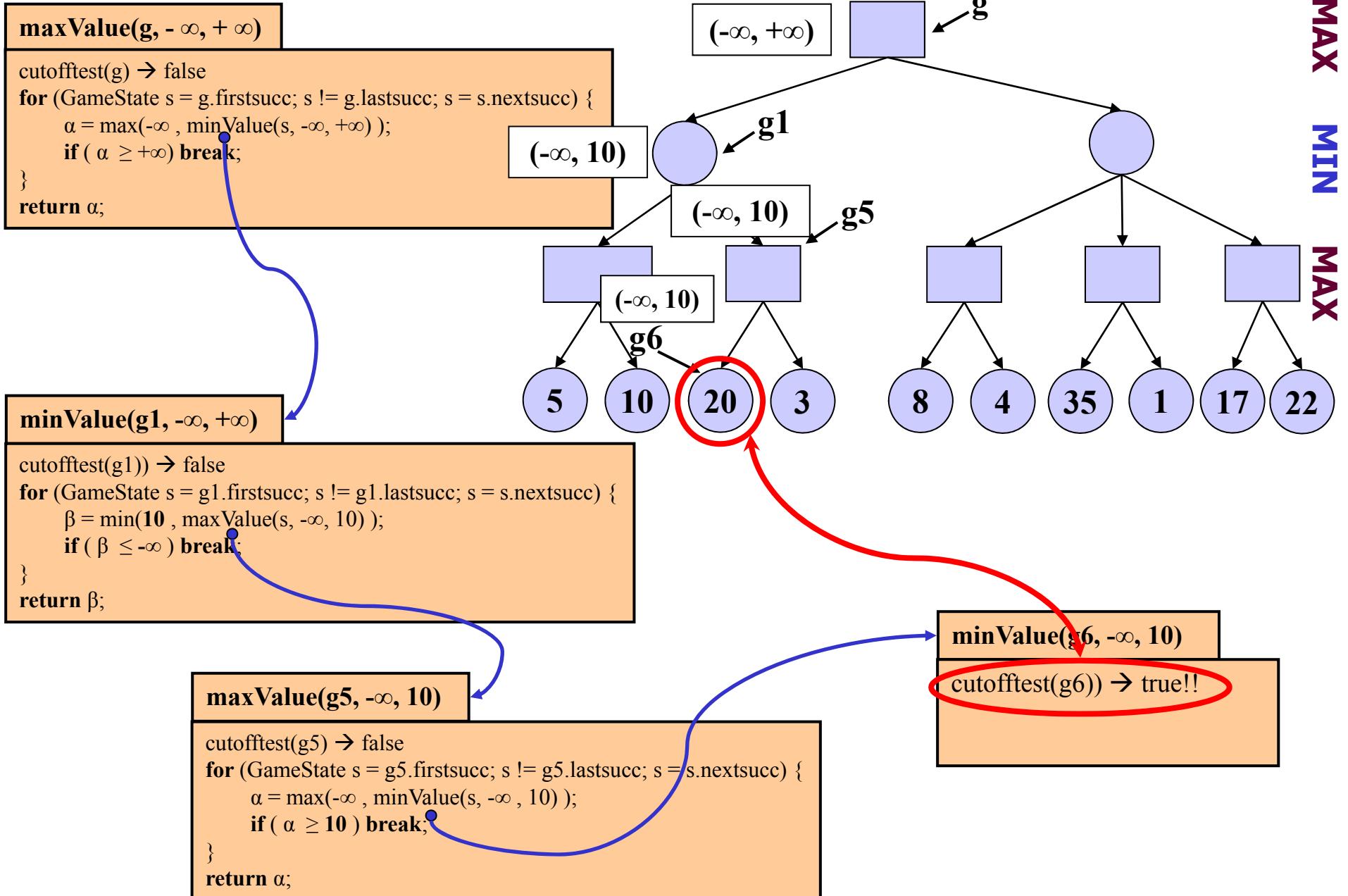
maxValue(g5, -∞, 10)

```
cutofftest(g5) → false
for (GameState s = g5.firstsucc; s != g5.lastsucc; s = s.nextsucc) {
    α = max(-∞, minValue(s, -∞, 10));
    if (α ≥ 10) break;
}
return α;
```

minValue(g6, -∞, 10)



MAX
MIN
MAX



maxValue(g, -∞, +∞)

```
cutofftest(g) → false
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {
    α = max(-∞, minValue(s, -∞, +∞));
    if (α ≥ +∞) break;
}
return α;
```

minValue(g1, -∞, +∞)

```
cutofftest(g1)) → false
for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {
    β = min(10, maxValue(s, -∞, 10));
    if (β ≤ -∞) break;
}
return β;
```

maxValue(g5, -∞, 10)

```
cutofftest(g5) → false
for (GameState s = g5.firstsucc; s != g5.lastsucc; s = s.nextsucc) {
    α = max(-∞, minValue(s, -∞, 10));
    if (α ≥ 10) break;
}
return α;
```

(-∞, +∞)

g

(-∞, 10)

(-∞, 10)

5

10

20

3

8

4

35

1

17

22

MAX

MIN

MAX

minValue(g6, -∞, 10)

```
cutofftest(g6)) → true!!
⇒ eval(g6) = 20
⇒ return 20
```

maxValue(g, -∞, +∞)

```
cutofftest(g) → false
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {
    α = max(-∞, minValue(s, -∞, +∞));
    if (α ≥ +∞) break;
}
return α;
```

minValue(g1, -∞, +∞)

```
cutofftest(g1)) → false
for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {
    β = min(10, maxValue(s, -∞, 10));
    if (β ≤ -∞) break;
}
return β;
```

maxValue(g5, -∞, 10)

```
cutofftest(g5) → false
for (GameState s = g5.firstsucc; s != g5.lastsucc; s = s.nextsucc) {
    α = max(-∞, minValue(s, -∞, 10));
    if (α ≥ 10) break;
}
return α;
```

(-∞, +∞)

g

g1

g5

5 10 20 3

8 4 35 1

17 22

MAX

MIN

MAX

minValue(g6, -∞, 10)

```
cutofftest(g6)) → true!!
⇒ eval(g6) = 20
⇒ return 20
```

maxValue(g, -∞, +∞)

```
cutofftest(g) → false
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {
    α = max(-∞, minValue(s, -∞, +∞));
    if (α ≥ +∞) break;
}
return α;
```

minValue(g1, -∞, +∞)

```
cutofftest(g1)) → false
for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {
    β = min(10, maxValue(s, -∞, 10));
    if (β ≤ -∞) break;
}
return β;
```

maxValue(g5, -∞, 10)

```
cutofftest(g5) → false
for (GameState s = g5.firstsucc; s != g5.lastsucc; s = s.nextsucc) {
    α = max(-∞, 20);
    if (α ≥ 10) break;
}
return α;
```

(-∞, +∞)

g

g1

g5

5 10 20 3

8 4 35 1

17 22

MAX
MIN
MAX

minValue(g6, -∞, 10)

```
cutofftest(g6)) → true!!
⇒ eval(g6) = 20
⇒ return 20
```

maxValue(g, -∞, +∞)

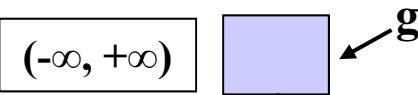
```
cutofftest(g) → false
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {
    α = max(-∞, minValue(s, -∞, +∞));
    if (α ≥ +∞) break;
}
return α;
```

minValue(g1, -∞, +∞)

```
cutofftest(g1) → false
for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {
    β = min(10, maxValue(s, -∞, 10));
    if (β ≤ -∞) break;
}
return β;
```

maxValue(g5, -∞, 10)

```
cutofftest(g5) → false
for (GameState s = g5.firstsucc; s != g5.lastsucc; s = s.nextsucc) {
    α = 20;
    if (α ≥ 10) break;
}
return α;
```



(-∞, 10)

(20, 10)

5 10

20 3

8 4

35 1

17 22

(-∞, 10)

(20, 10)

5 10

20 3

8 4

35 1

17 22

MAX
MIN
MAX

maxValue(g, -∞, +∞)

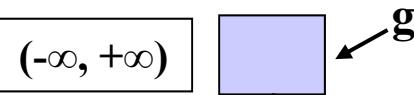
```
cutofftest(g) → false
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {
    α = max(-∞, minValue(s, -∞, +∞));
    if (α ≥ +∞) break;
}
return α;
```

minValue(g1, -∞, +∞)

```
cutofftest(g1) → false
for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {
    β = min(10, maxValue(s, -∞, 10));
    if (β ≤ -∞) break;
}
return β;
```

maxValue(g5, -∞, 10)

```
cutofftest(g5) → false
for (GameState s = g5.firstsucc; s != g5.lastsucc; s = s.nextsucc) {
    α = 20;
    if (20 ≥ 10) break;
}
return α; true!
```



(-∞, 10)

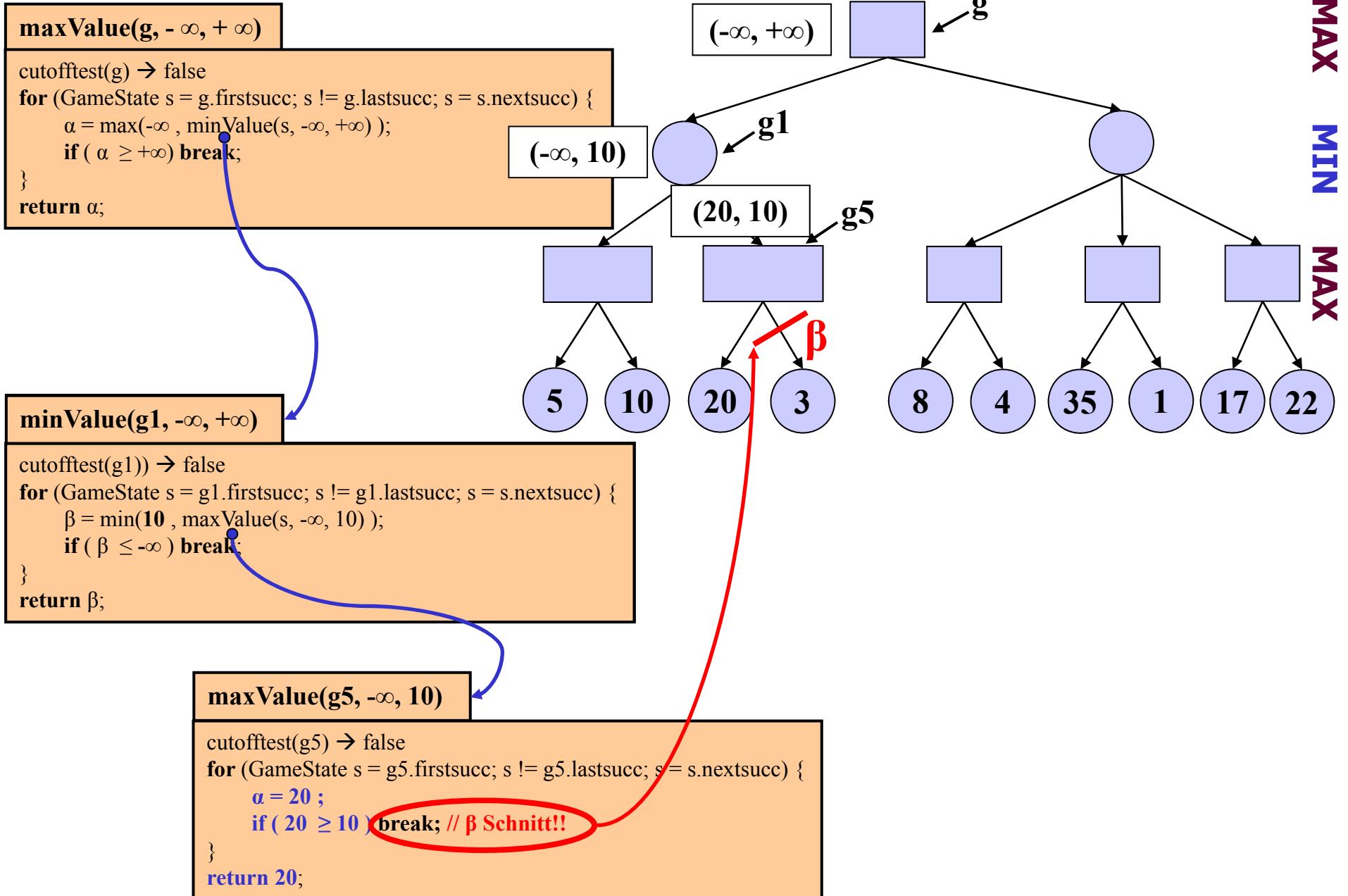
(20, 10)

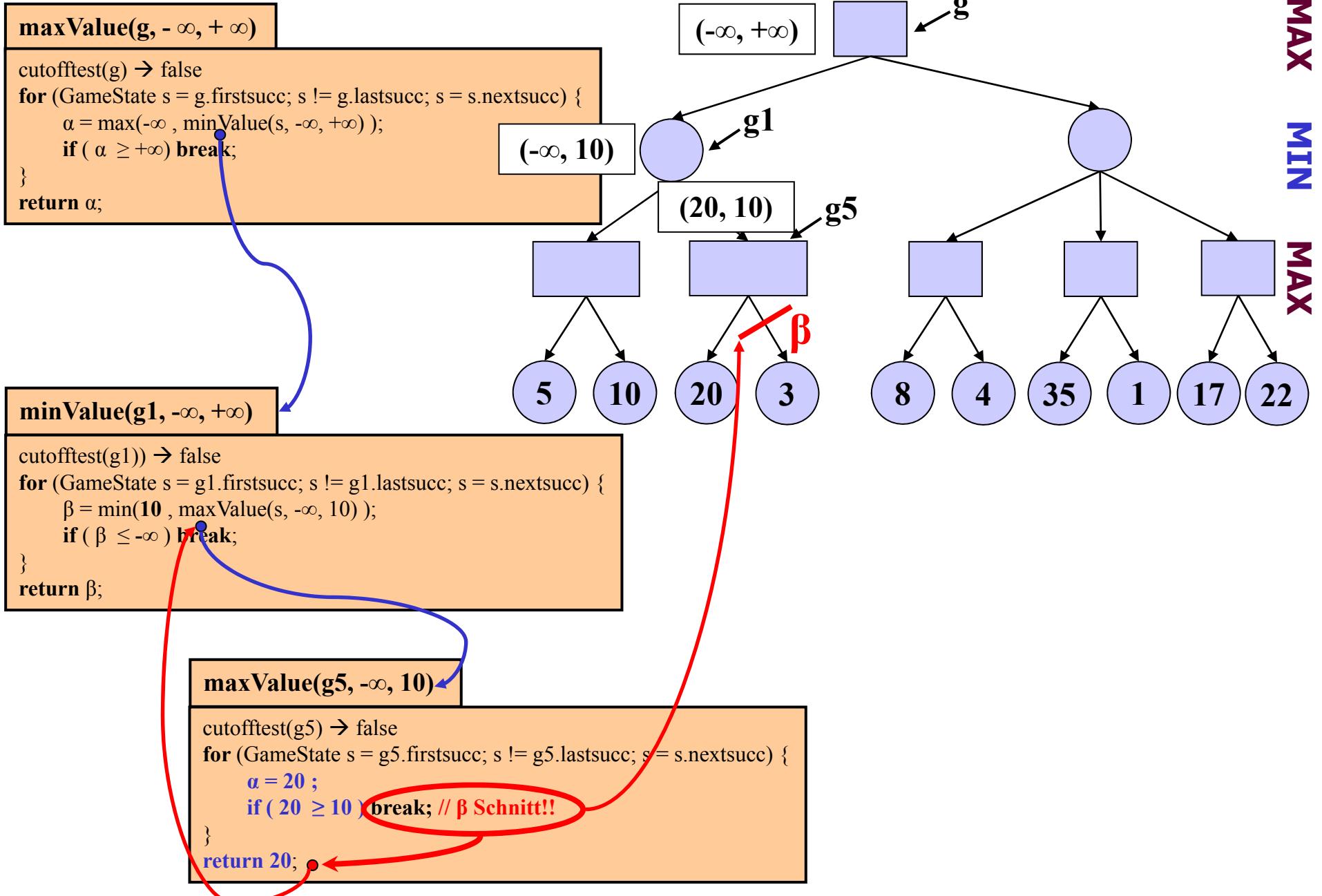
5 10

20 3

8 4 35 1 17 22

MAX MIN MAX





maxValue(g, -∞, +∞)

```
cutofftest(g) → false
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {
    α = max(-∞, minValue(s, -∞, +∞));
    if (α ≥ +∞) break;
}
return α;
```

minValue(g1, -∞, +∞)

```
cutofftest(g1)) → false
for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {
    β = min(10, 20);
    if (β ≤ -∞) break;
}
return β;
```

maxValue(g5, -∞, 10)

```
cutofftest(g5) → false
for (GameState s = g5.firstsucc; s != g5.lastsucc; s = s.nextsucc) {
    α = 20;
    if (20 ≥ 10) break; // β Schnitt!!
}
return 20;
```

(-∞, +∞)

g1

(20, 10)

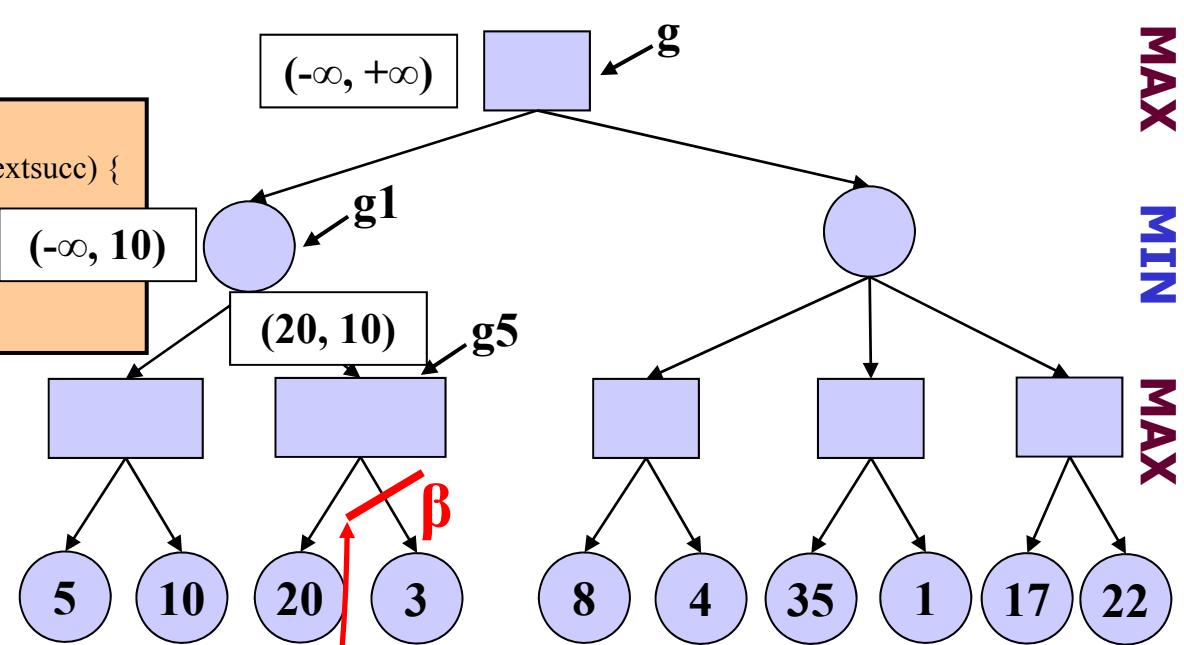
g5

β

MAX

MIN

MAX



maxValue(g, -∞, +∞)

```
cutofftest(g) → false
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {
    α = max(-∞, minValue(s, -∞, +∞));
    if (α ≥ +∞) break;
}
return α;
```

minValue(g1, -∞, +∞)

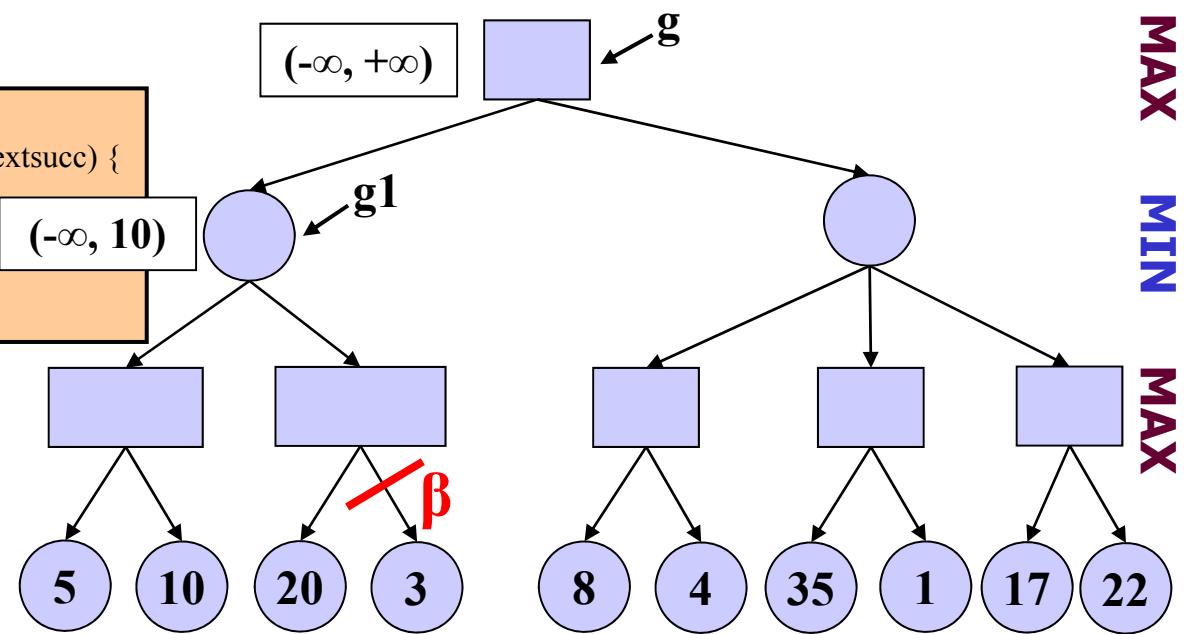
```
cutofftest(g1) → false
for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {
    β = 10;
    if (10 ≤ -∞) break;
}
return β;
```

(-∞, +∞)

g1

β

MAX
MIN
MAX



```
maxValue(g, -∞, +∞)
```

```
cutofftest(g) → false
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {
    α = max(-∞, minValue(s, -∞, +∞));
    if (α ≥ +∞) break;
}
return α;
```

```
(-∞, +∞)
```

```
g
```

MAX

MIN

MAX

```
minValue(g1, -∞, +∞)
```

```
cutofftest(g1) → false
for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {
    β = 10;
    if (10 ≤ -∞) break;
}
return β;
```

```
(-∞, 10)
```

```
g1
```

```
5 10
```

```
20 3
```

```
8 4
```

```
35 1
```

```
17 22
```

β

Red circle highlights the condition $if (10 \leq -\infty) break;$ which is false because $10 > -\infty$.

```
maxValue(g, -∞, +∞)
```

```
cutofftest(g) → false
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {
    α = max(-∞, minValue(s, -∞, +∞));
    if (α ≥ +∞) break;
}
return α;
```

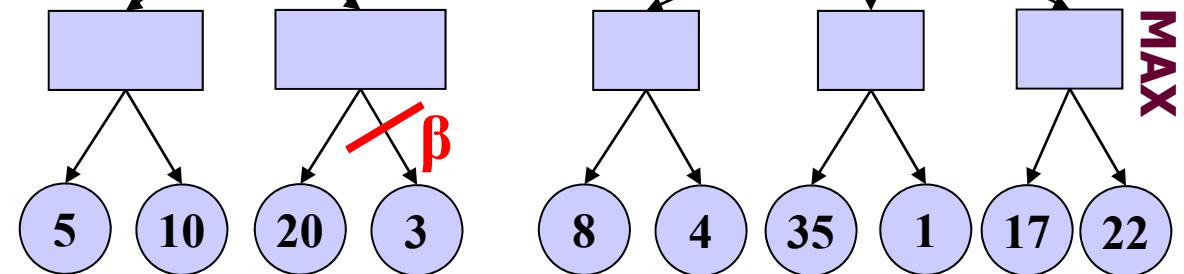
```
minValue(g1, -∞, +∞)
```

```
cutofftest(g1) → false
for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {
    β = 10;
    if (10 ≤ -∞) break;
}
return β;
```

(-∞, +∞) \xrightarrow{g}

(-∞, 10) $\xrightarrow{g_1}$

MAX
MIN
MAX



g1 has no more successors!

maxValue(g , $-\infty$, $+\infty$)

```
cutofftest( $g$ ) → false
for (GameState s =  $g$ .firstsucc; s !=  $g$ .lastsucc; s = s.nextsucc) {
     $\alpha$  = max( $-\infty$ , minValue(s,  $-\infty$ ,  $+\infty$ ));
    if ( $\alpha \geq +\infty$ ) break;
}
return  $\alpha$ ;
```

minValue(g_1 , $-\infty$, $+\infty$)

```
cutofftest( $g_1$ ) → false
for (GameState s =  $g_1$ .firstsucc; s !=  $g_1$ .lastsucc; s = s.nextsucc) {
     $\beta$  = 10;
    if ( $10 \leq -\infty$ ) break;
}
return 10;
```

$(-\infty, +\infty)$

g

MAX

MIN

MAX

$(-\infty, 10)$

5

10

20

3

8

4

35

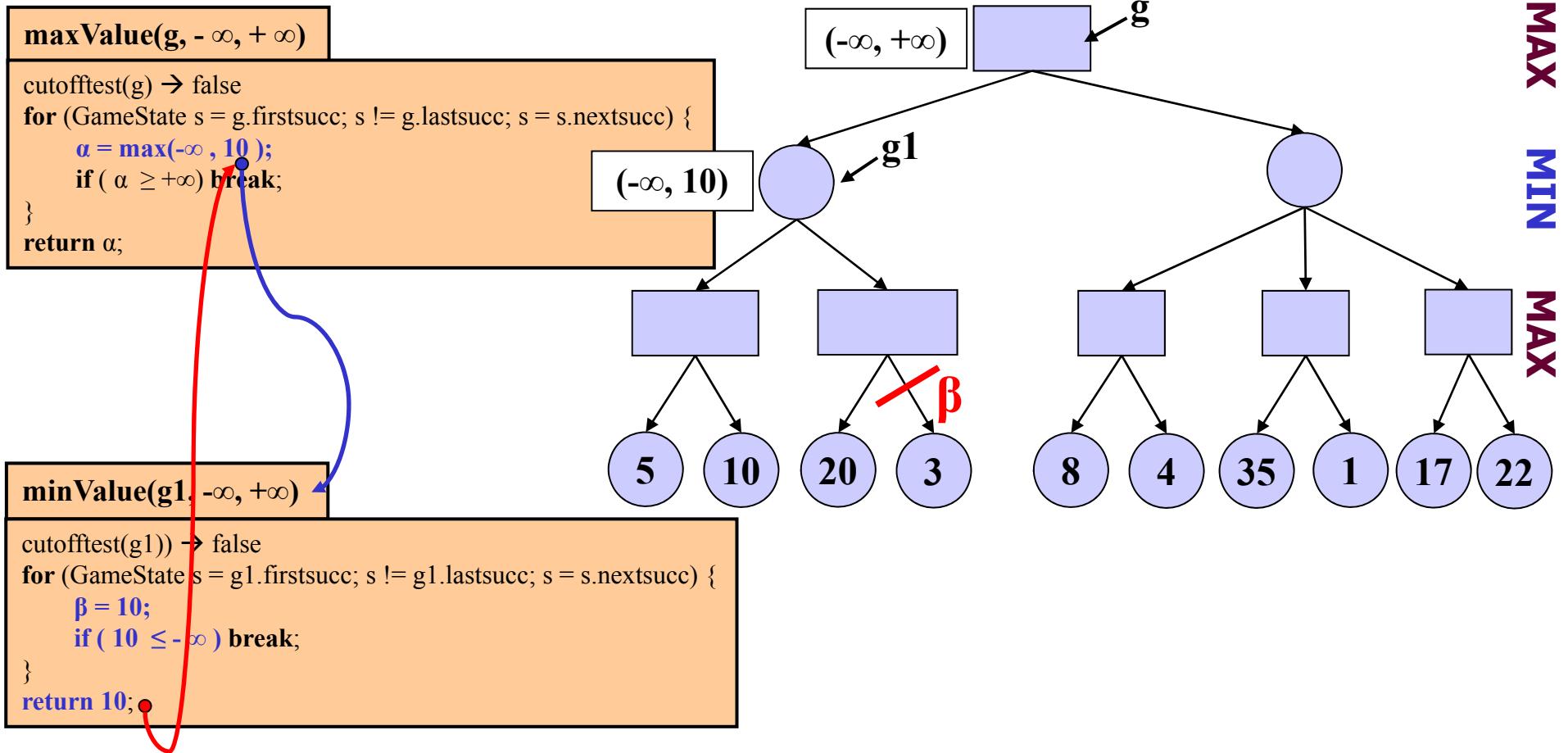
1

17

22

β

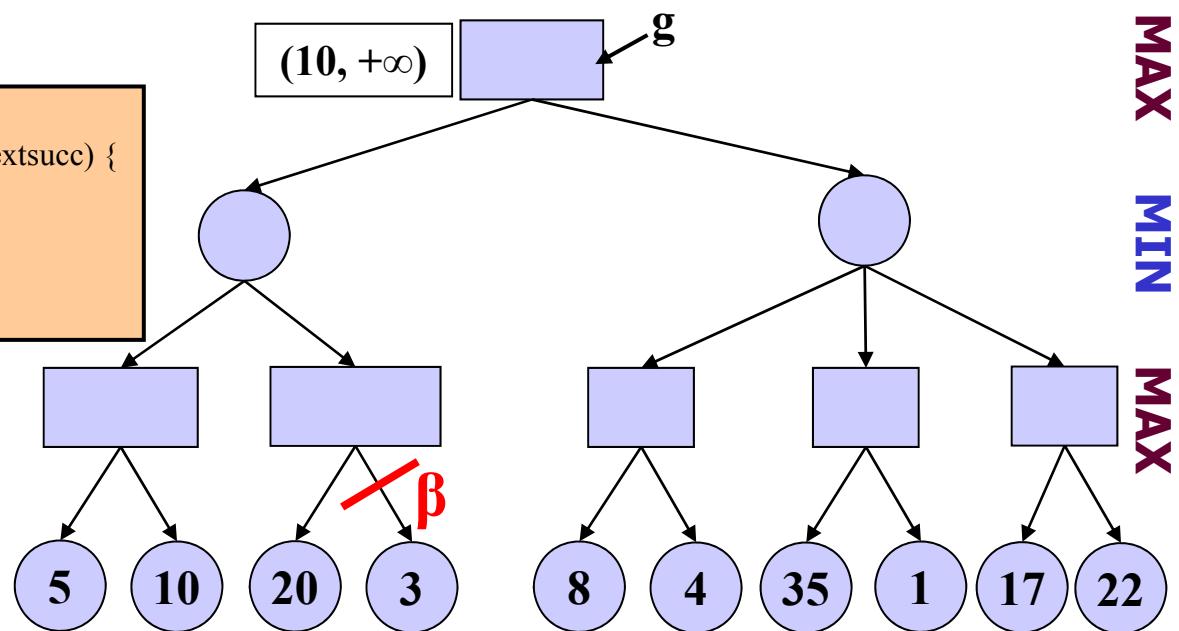
g1 has no more successors!



```
maxValue(g, -∞, +∞)
```

```
cutofftest(g) → false  
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {  
    α = 10;  
    if (α ≥ +∞) break;  
}  
return α;
```

(10, +∞) 



maxValue(g, -∞, +∞)

```
cutofftest(g) → false  
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {  
    a = 10;  
    if ( 10 ≥ +∞) break;  
}  
return α; false!
```

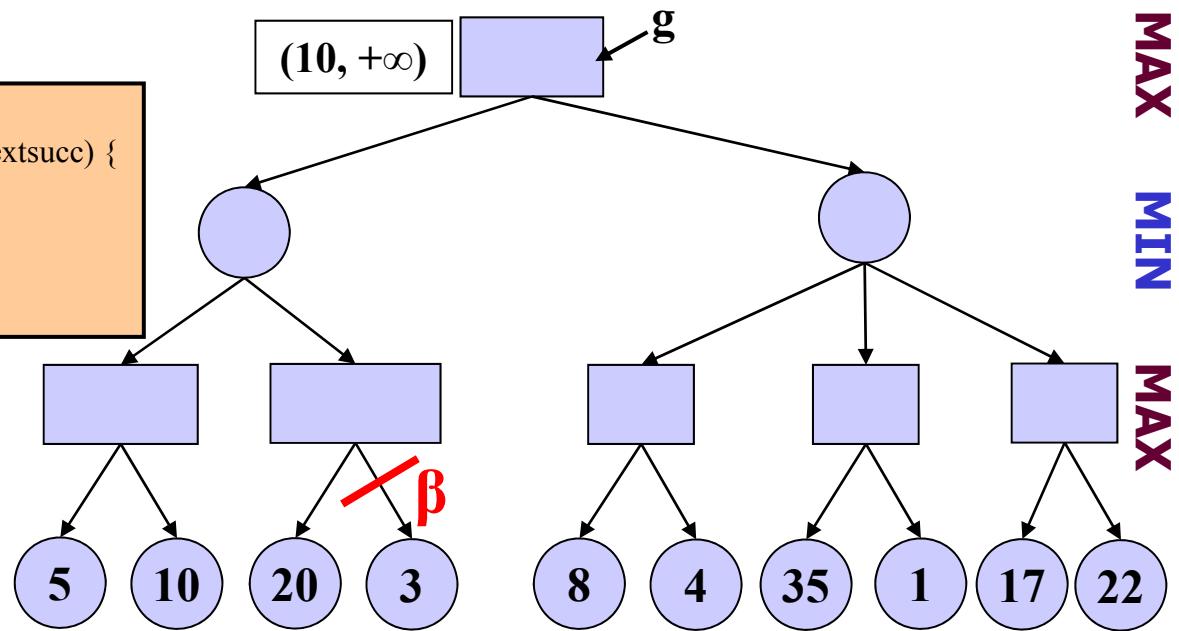
(10, +∞)



MAX

MIN

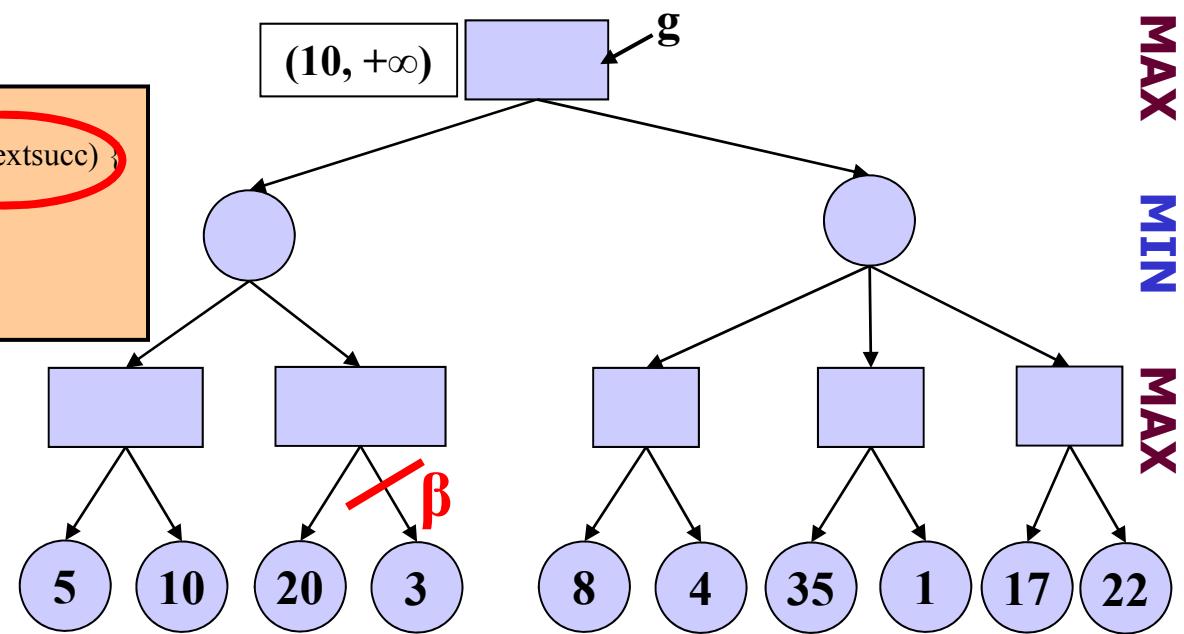
MAX

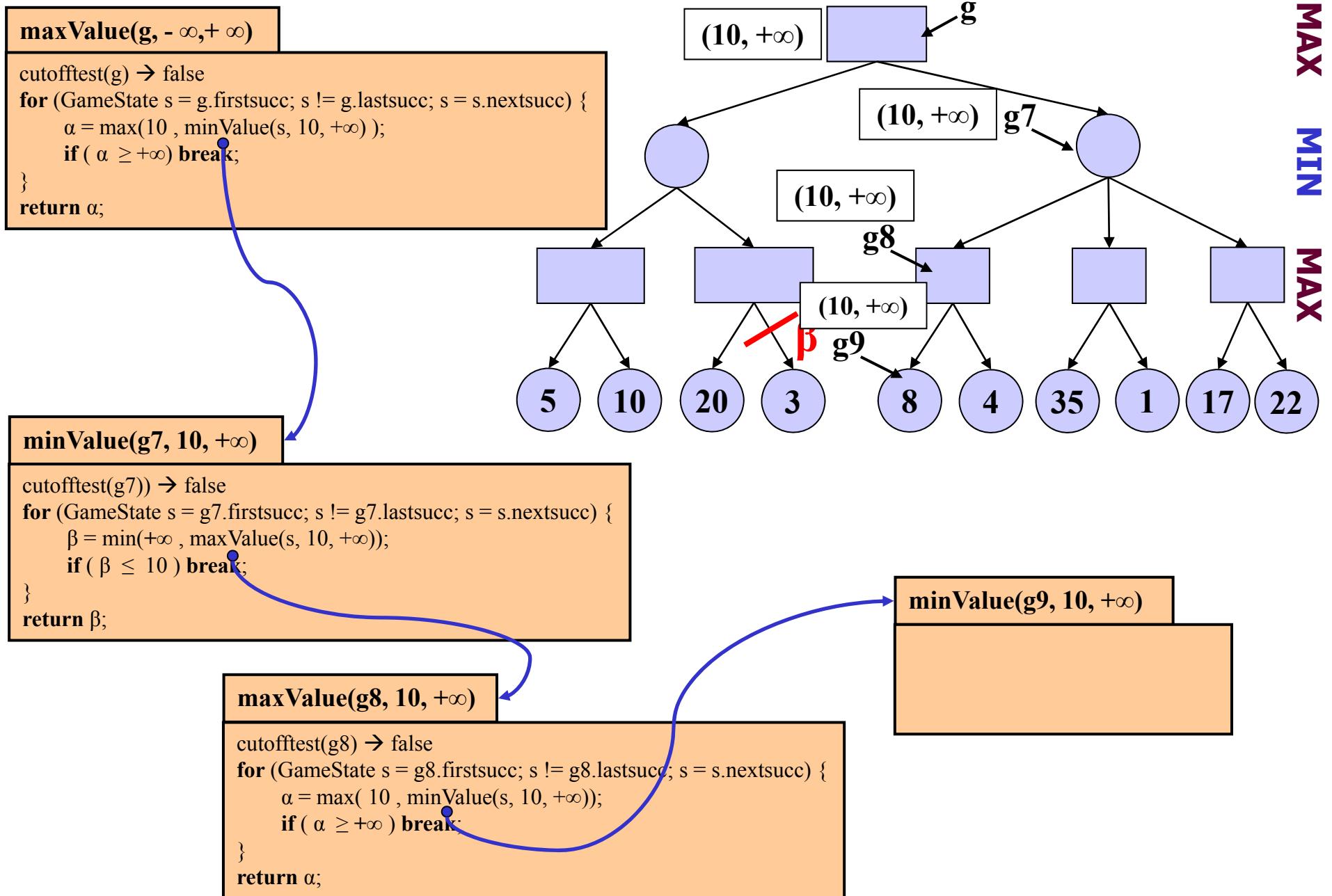


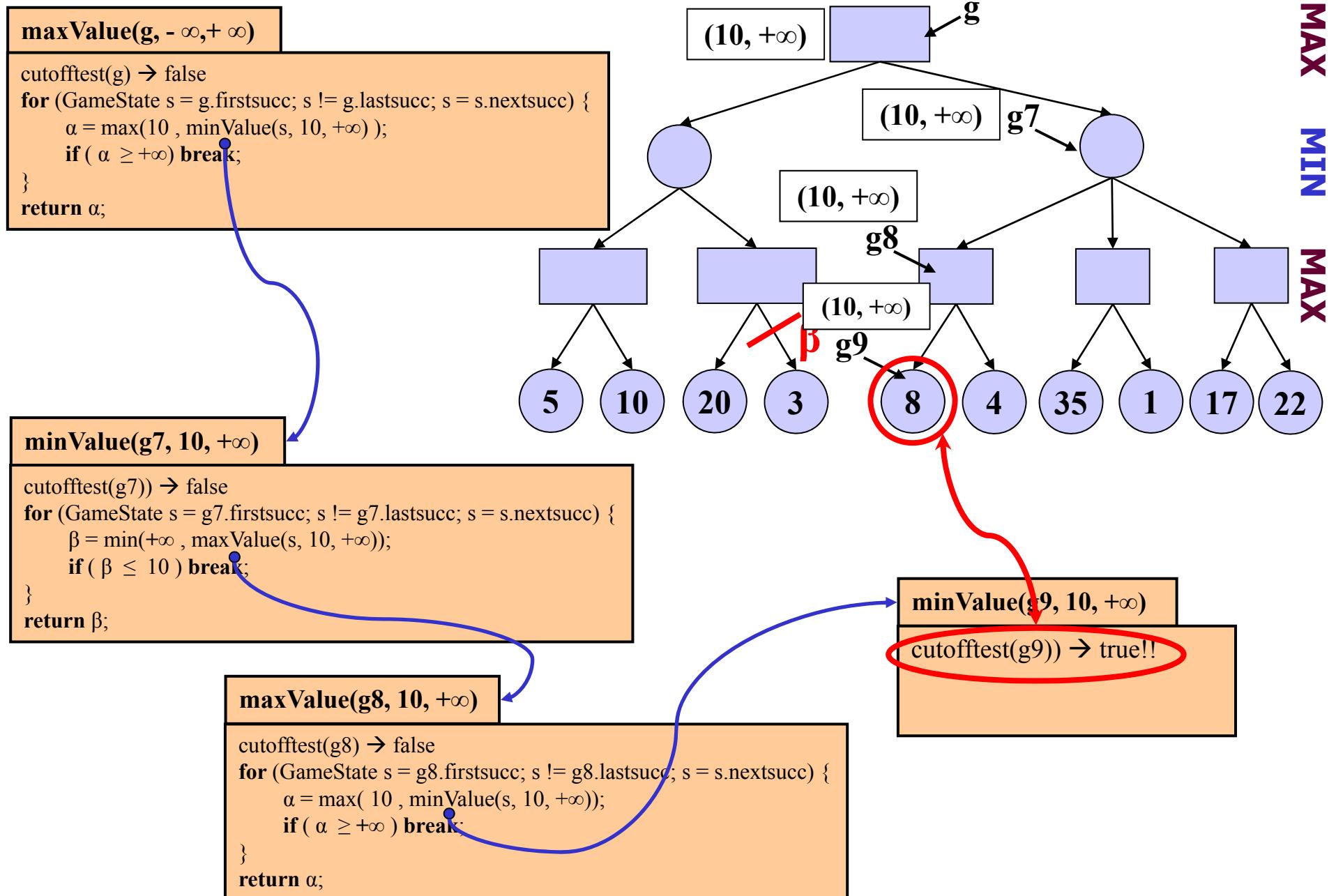
```
maxValue(g, -∞, +∞)
```

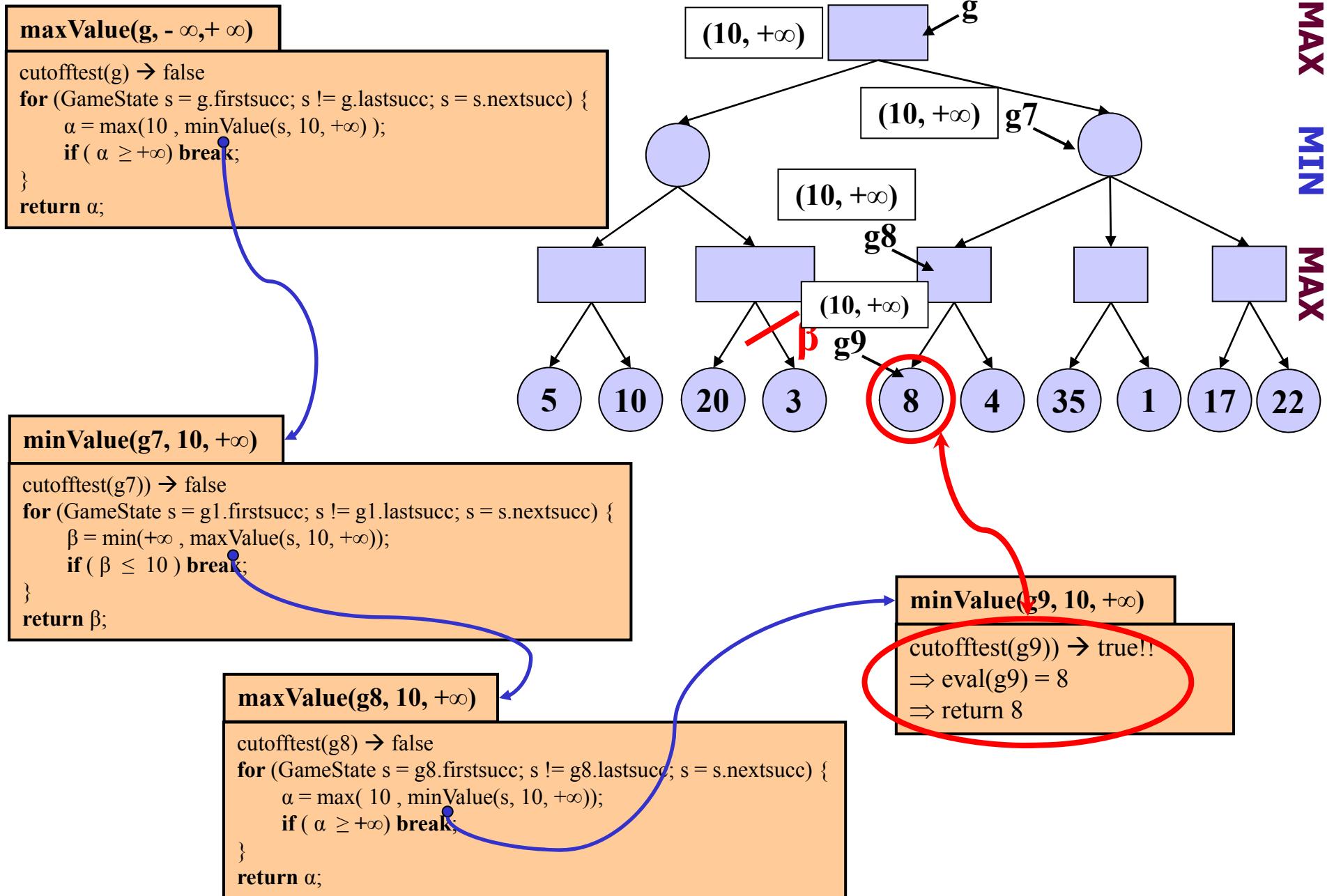
```
cutofftest(g) → false  
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {  
    α = max(10, minValue(s, 10, +∞));  
    if (α ≥ +∞) break;  
}  
return α;
```

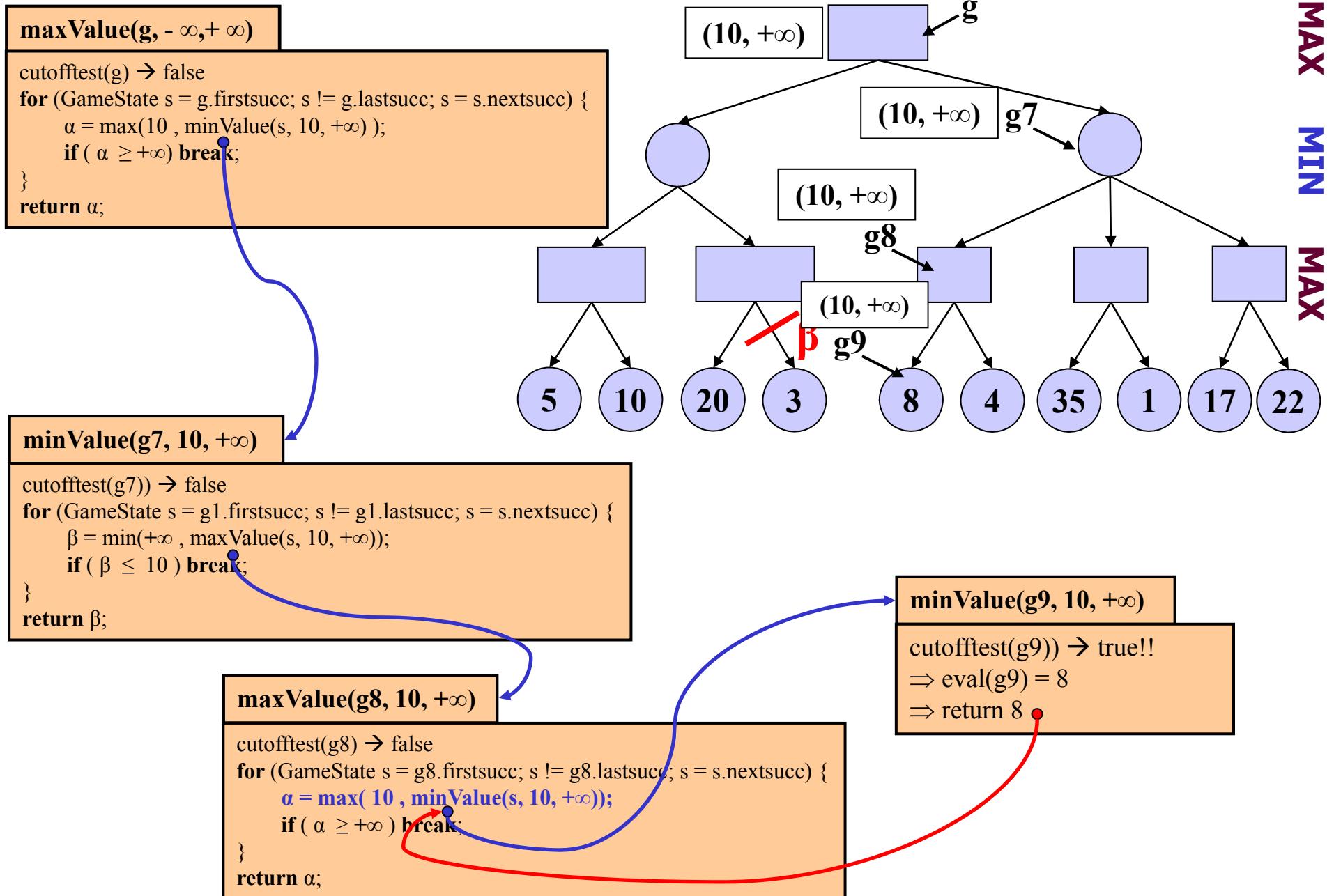
(10, +∞)  g

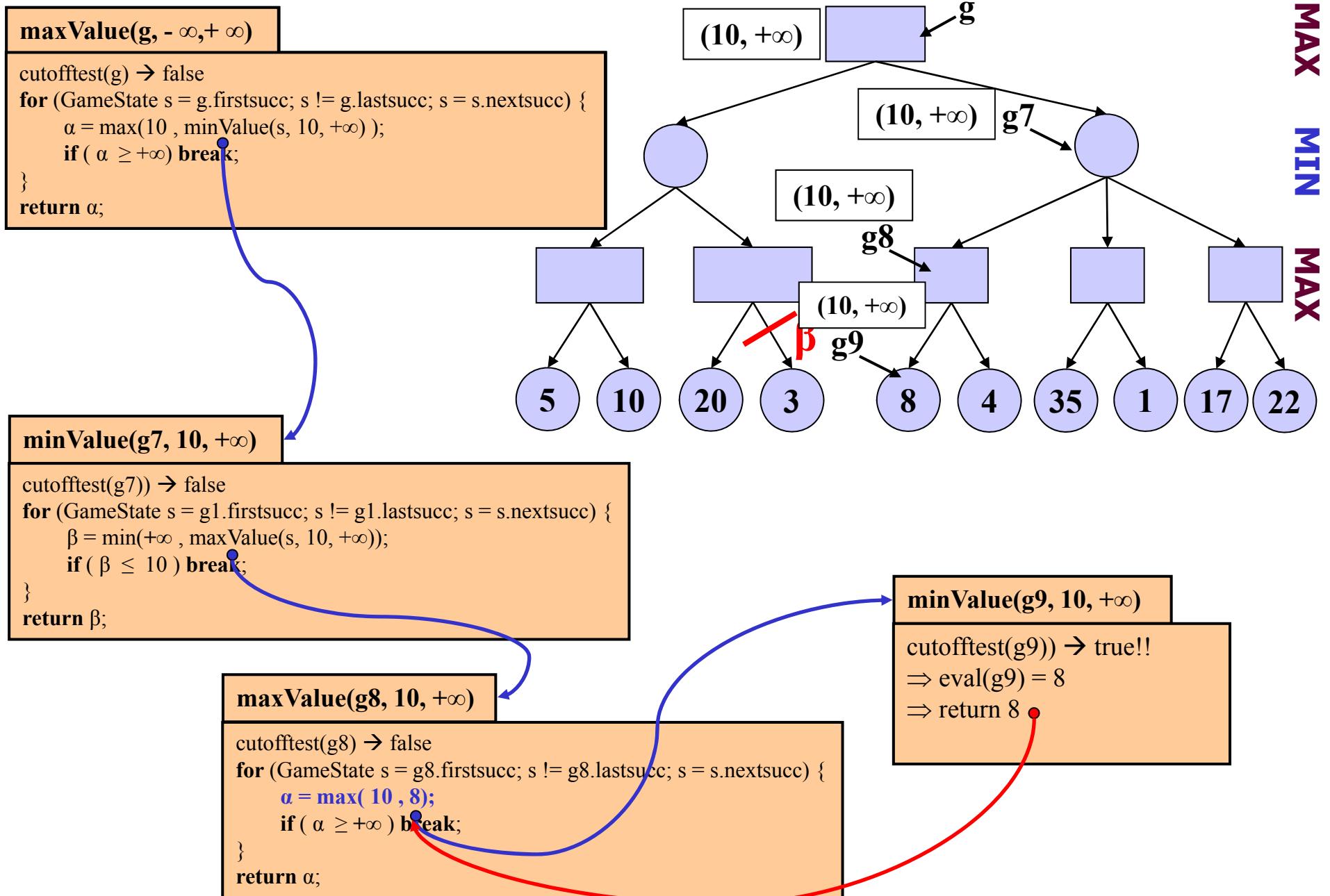


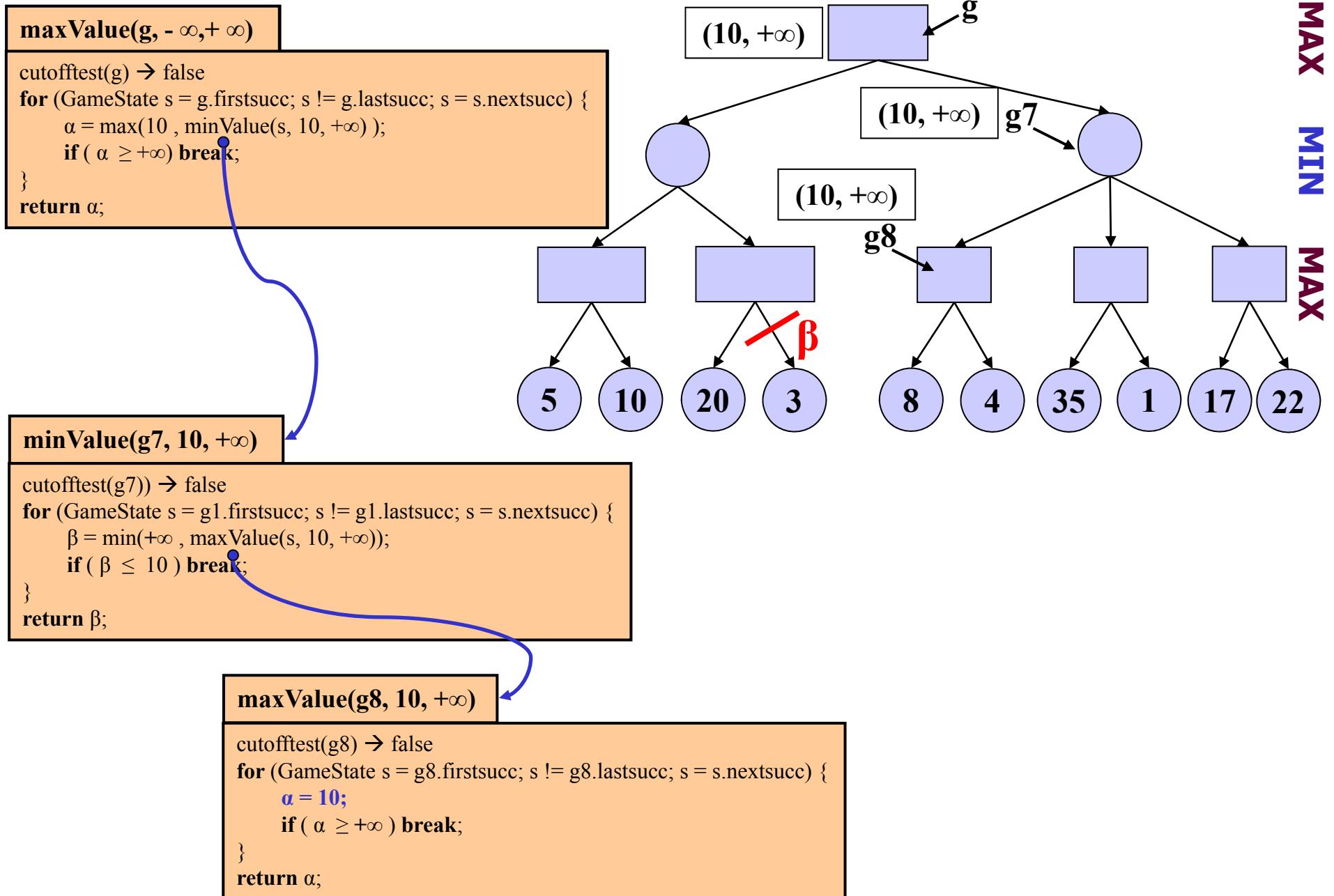


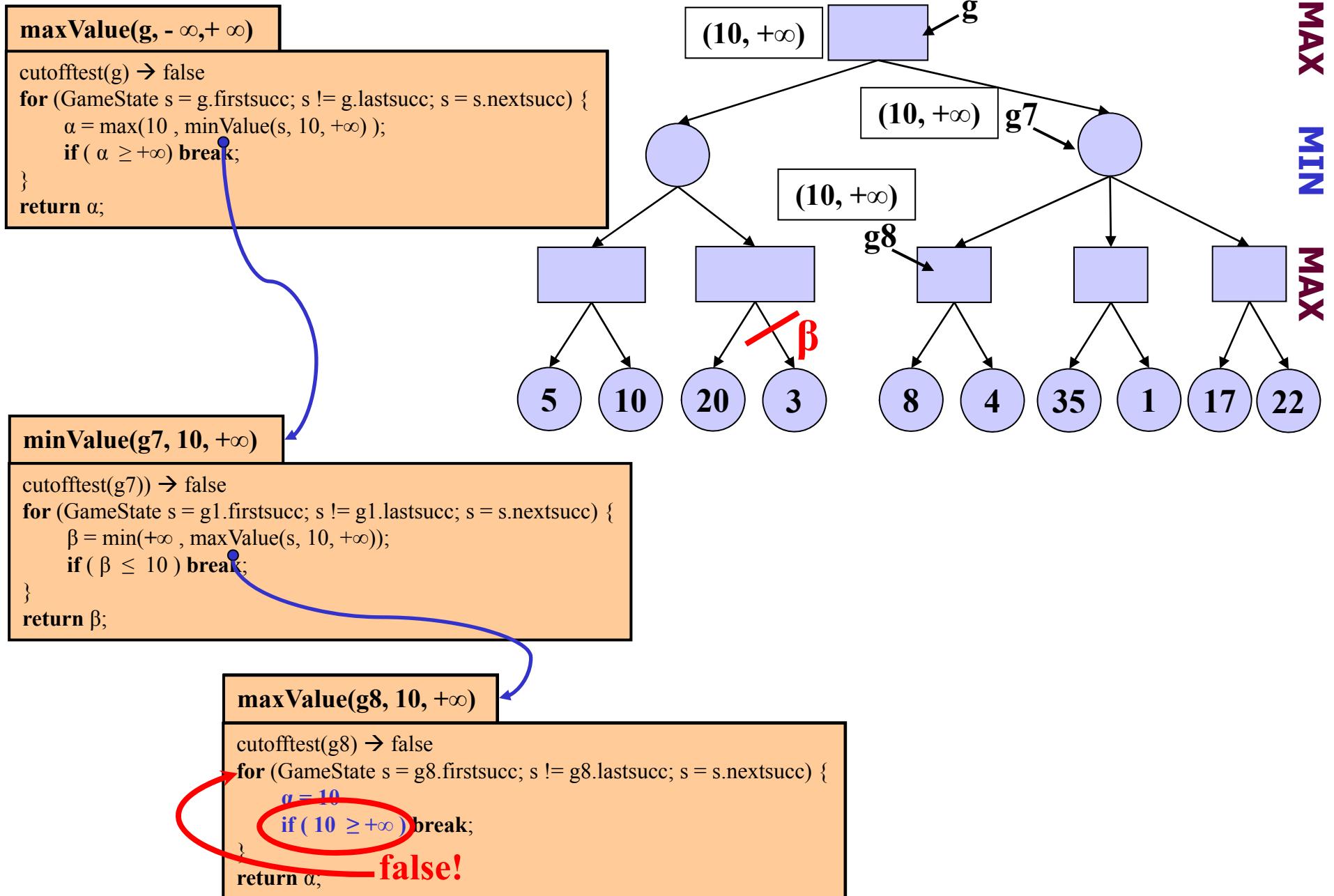


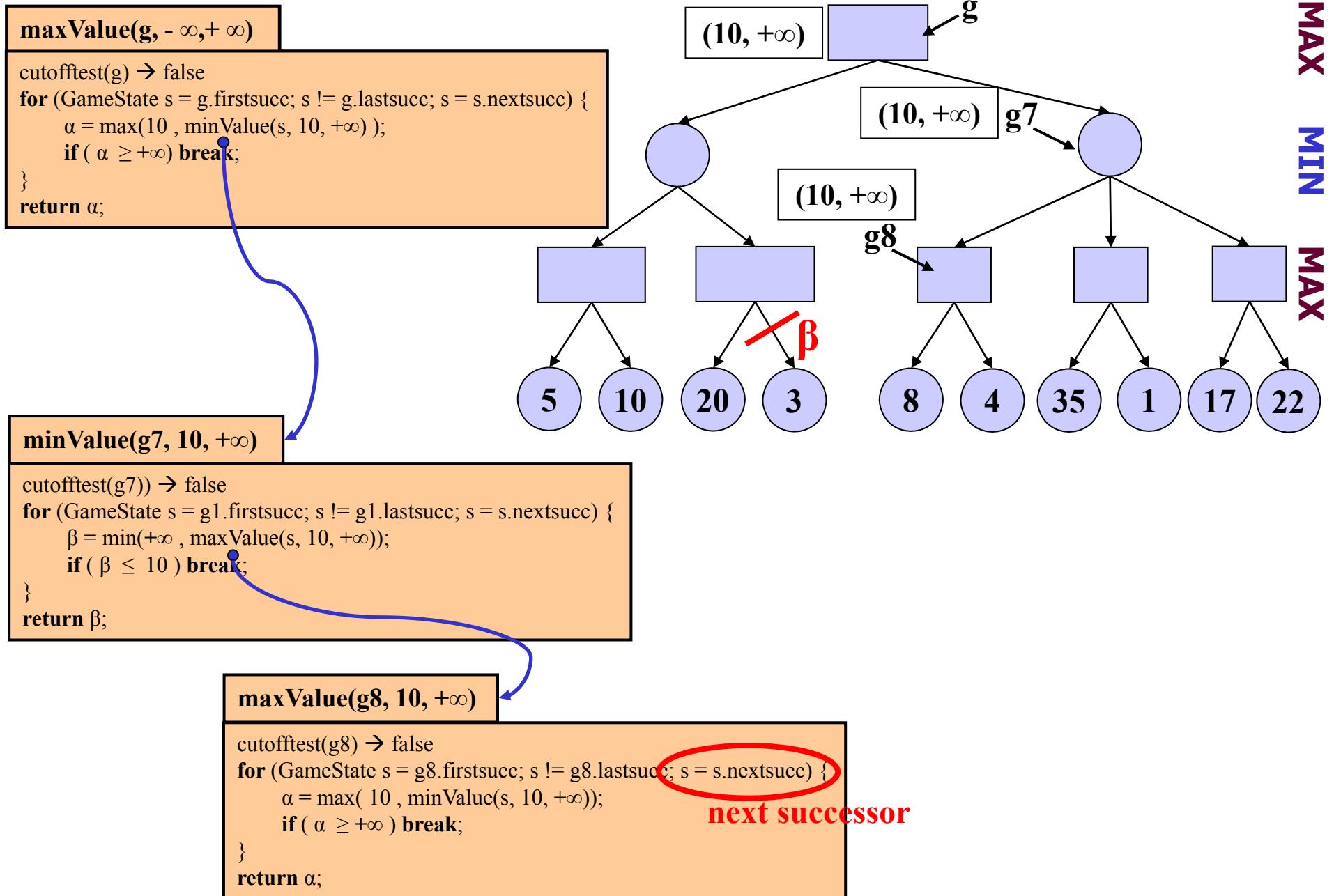


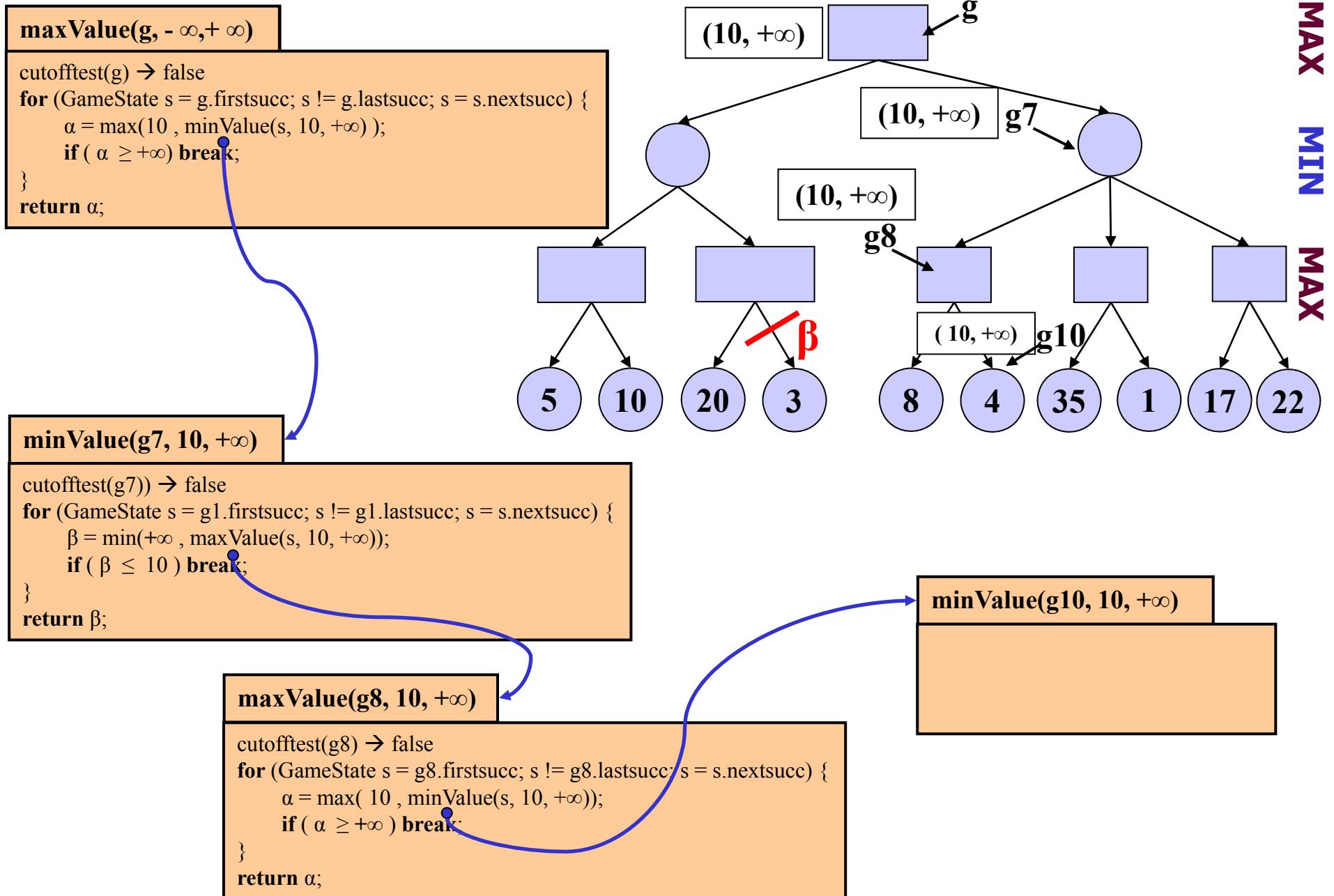


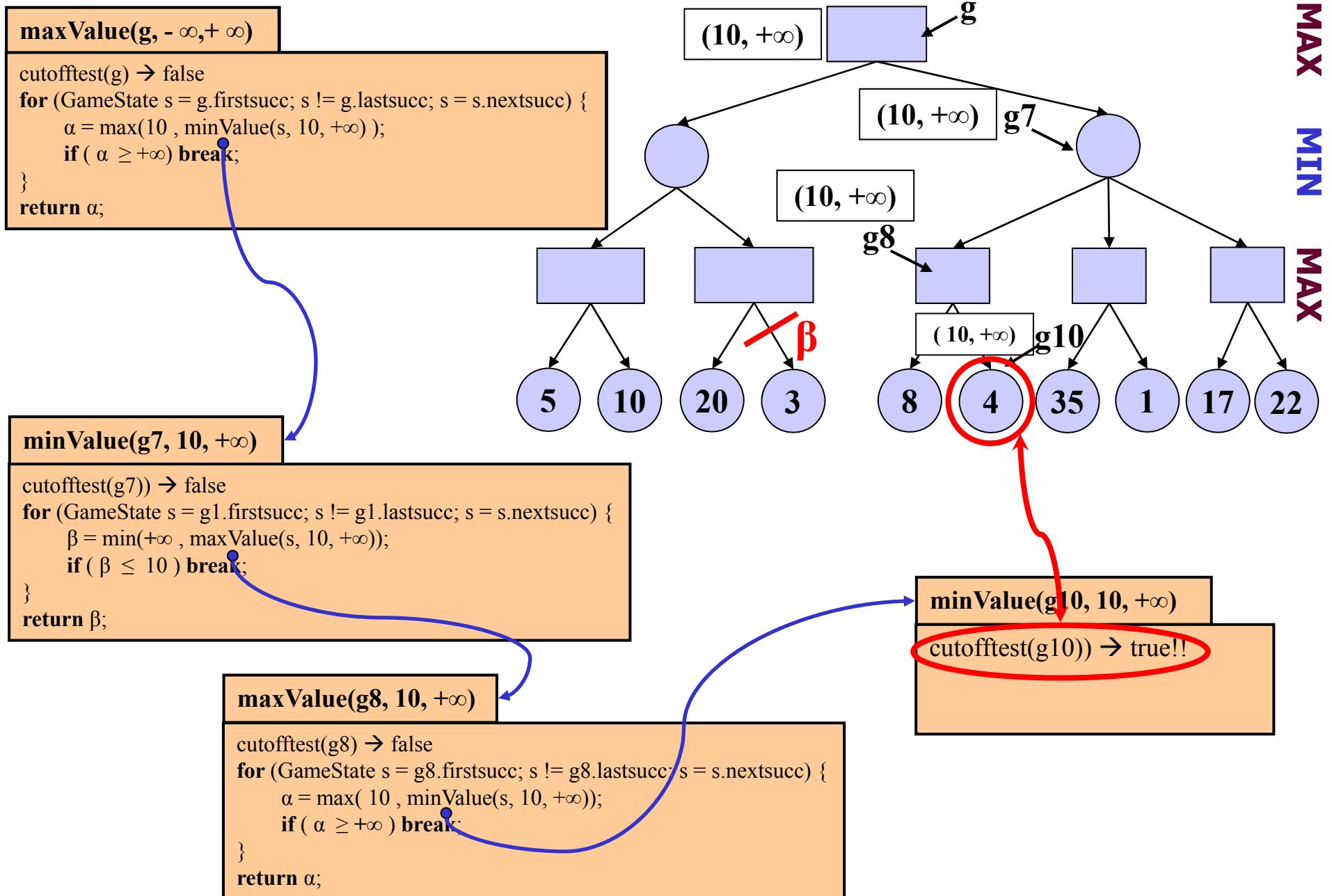


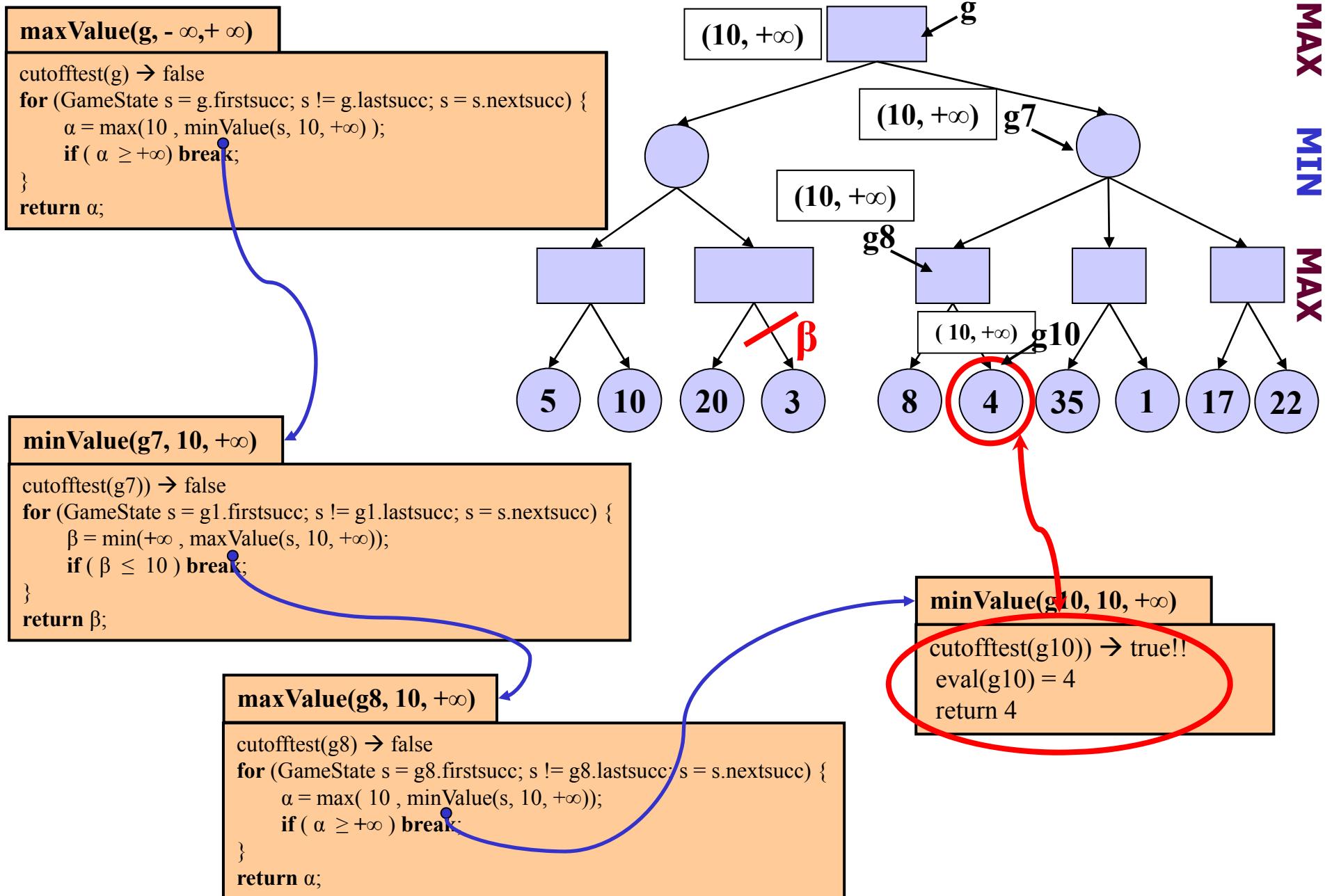


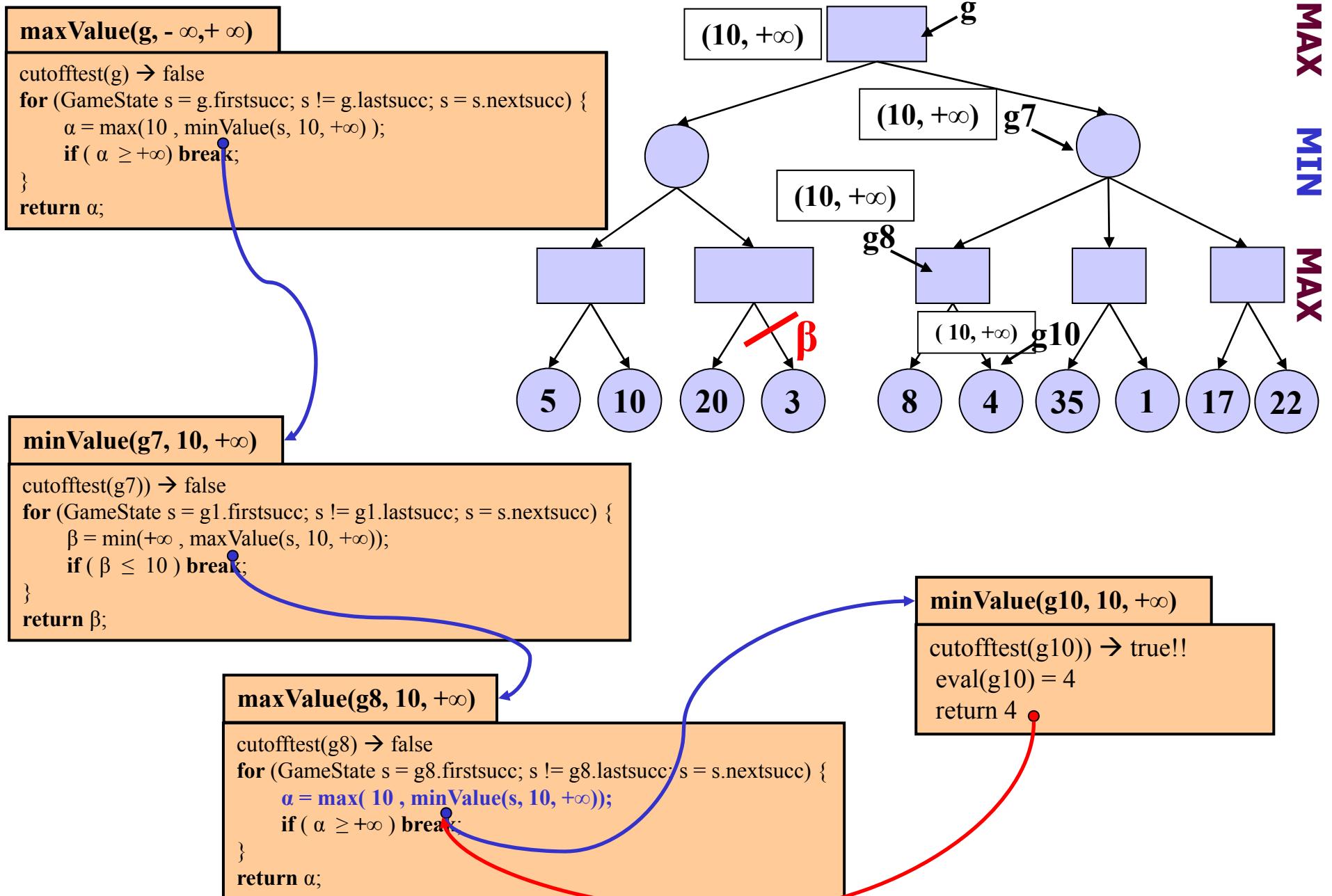


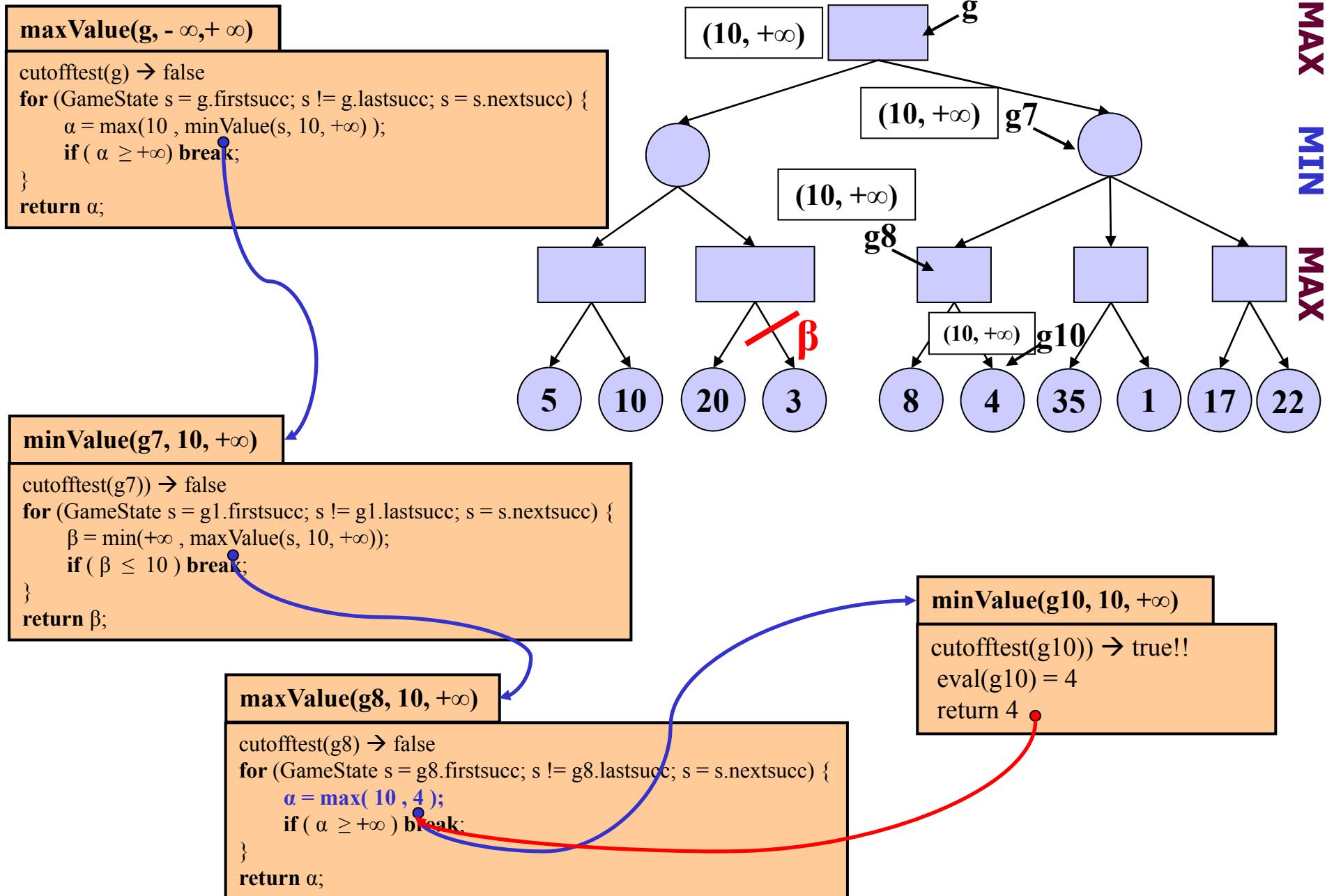


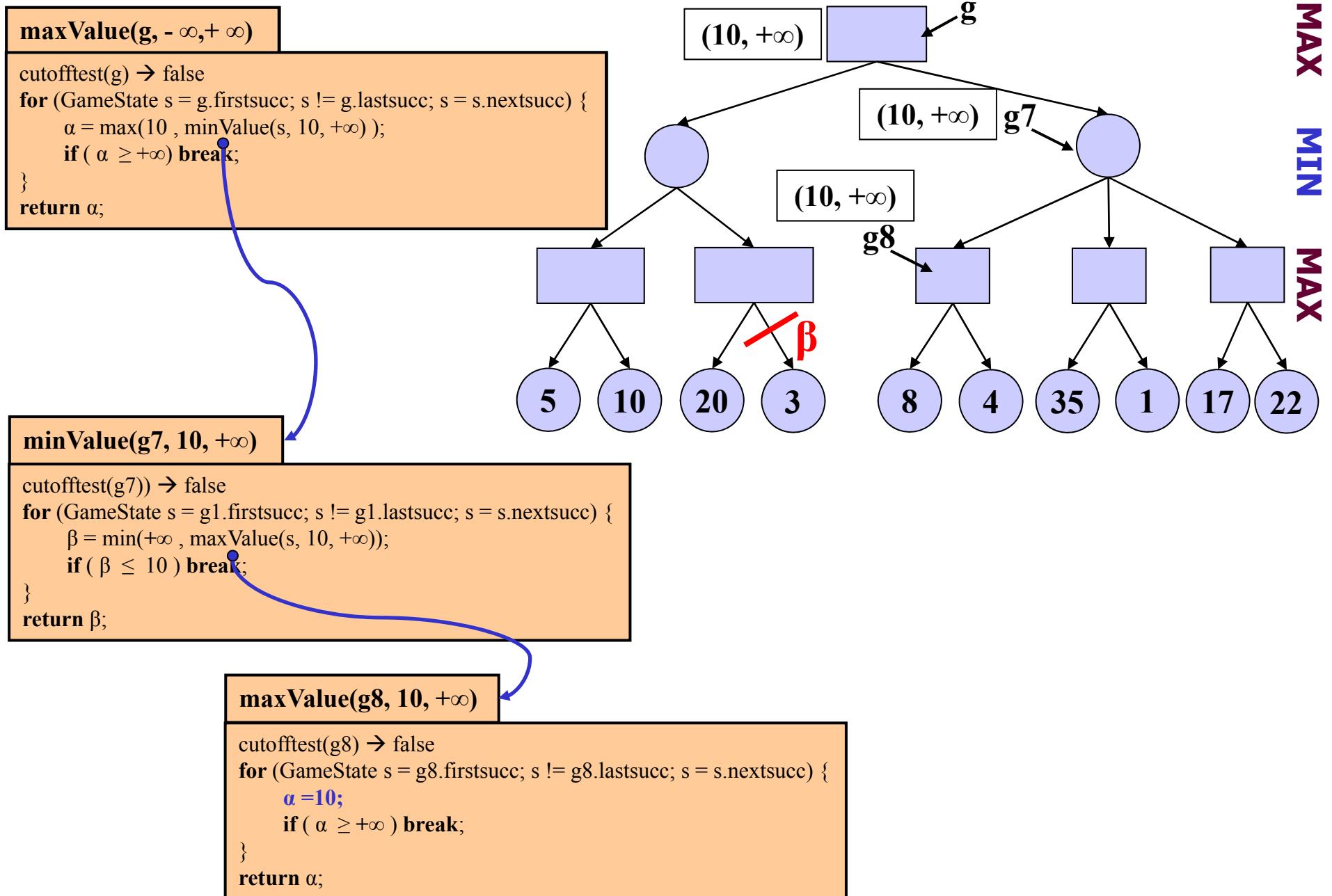


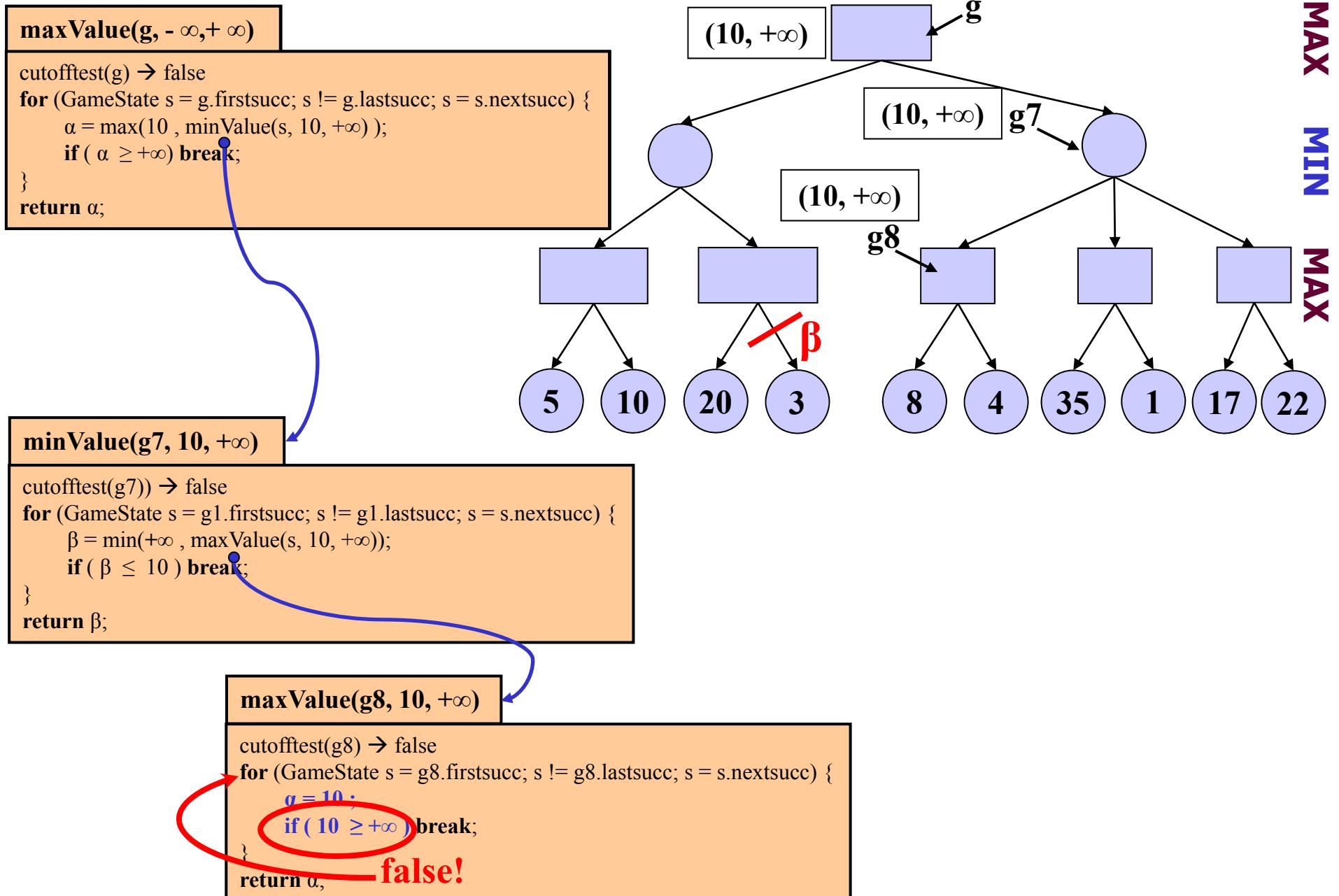


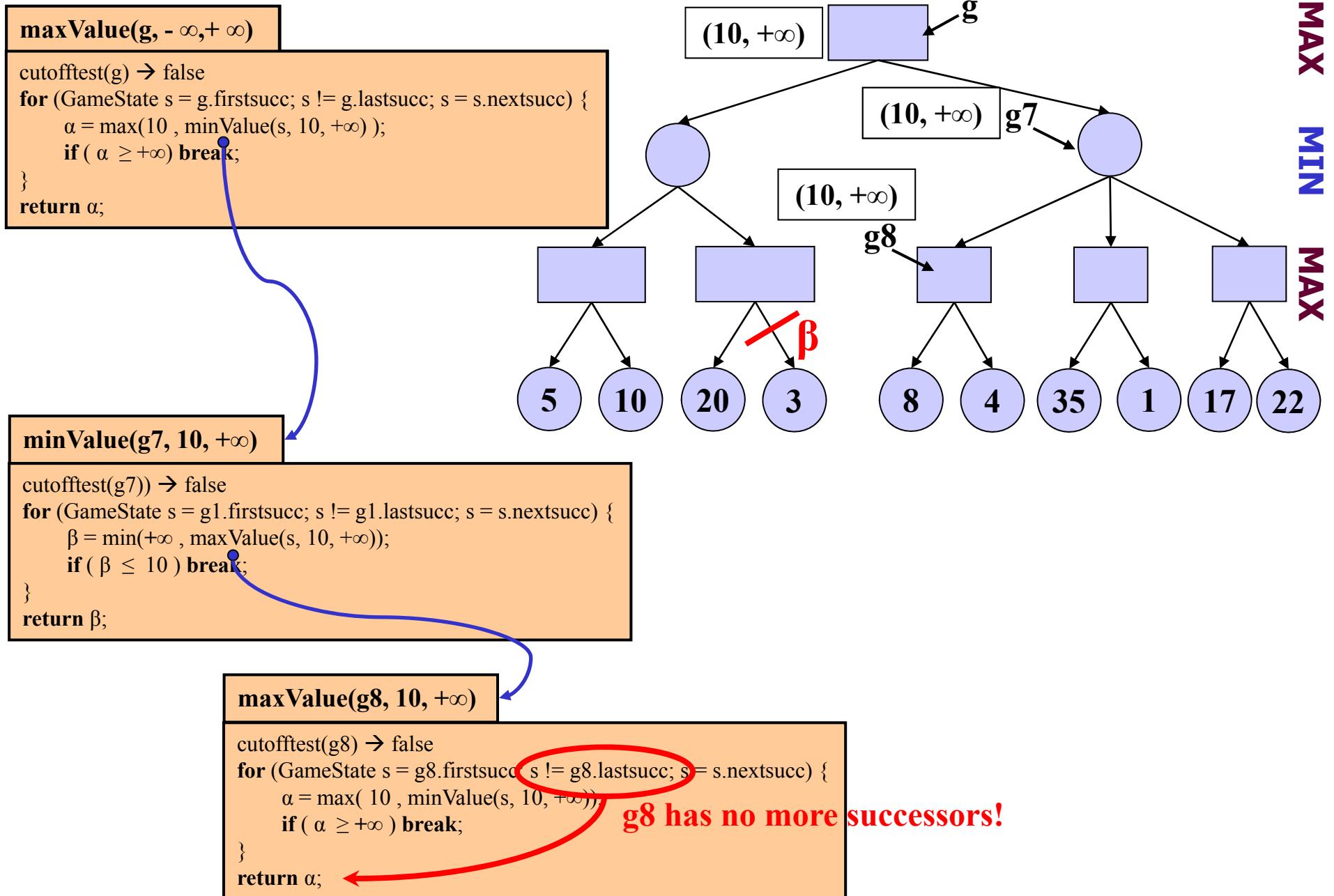


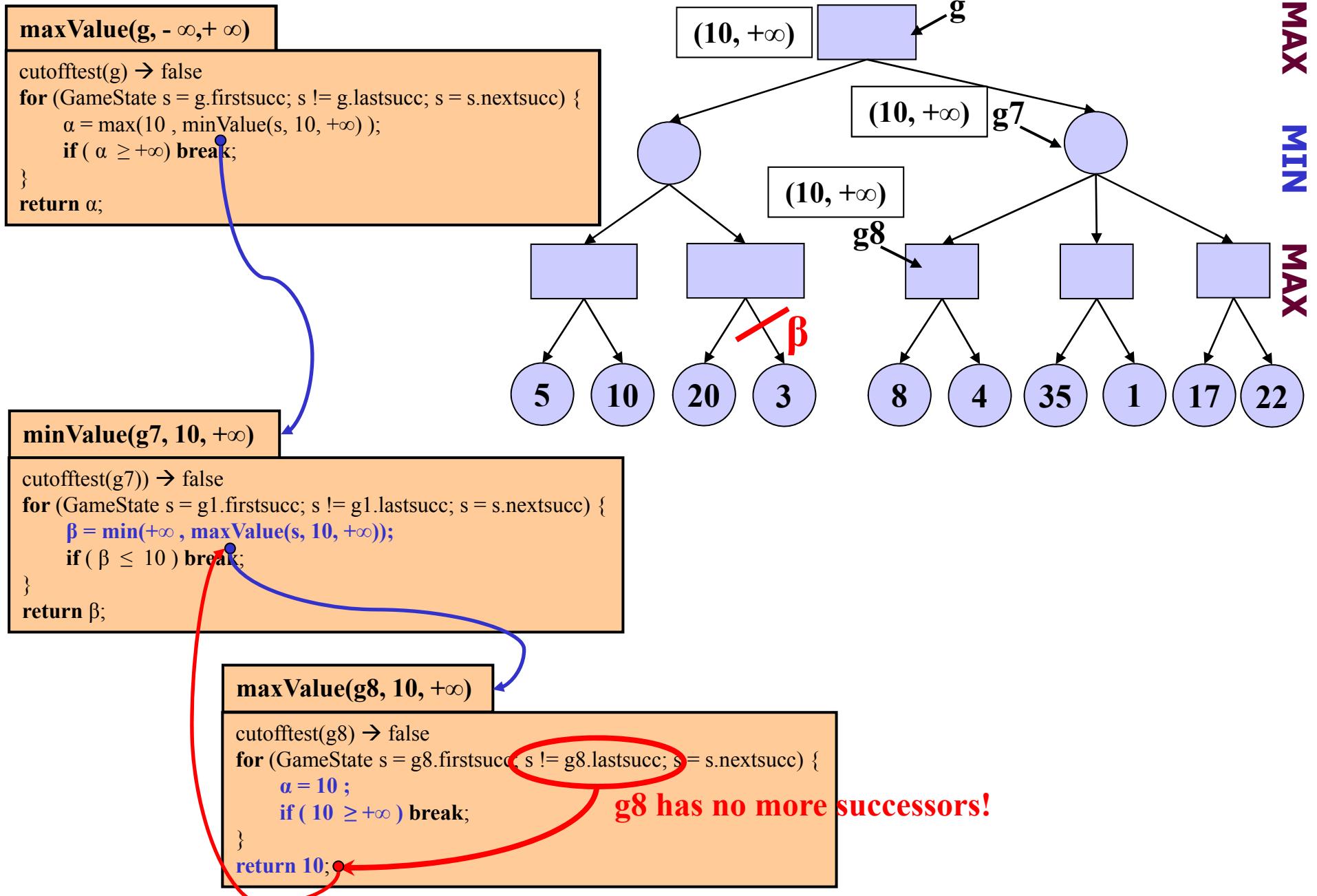










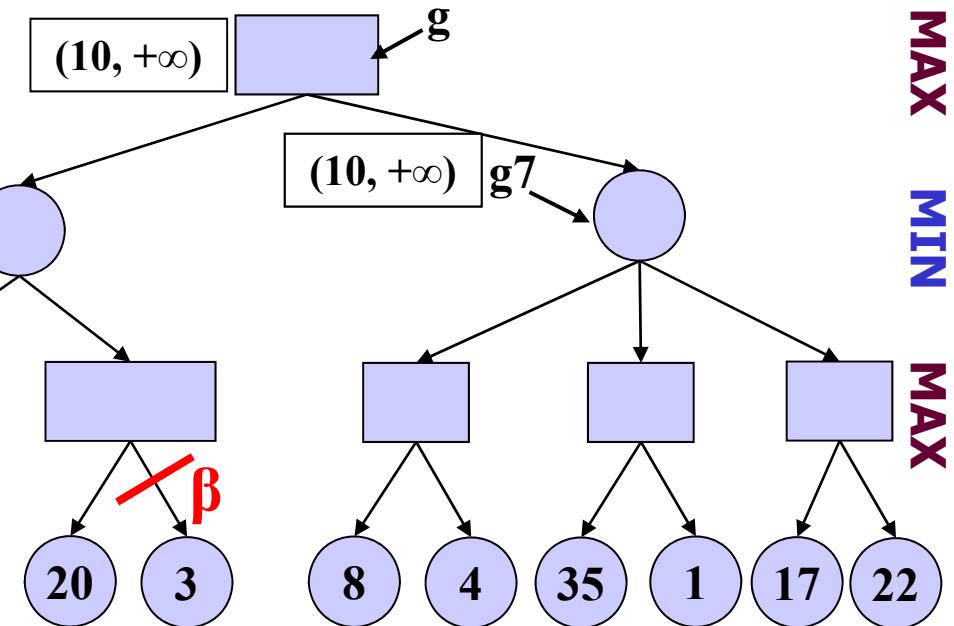


```
maxValue(g, -∞, +∞)
```

```
cutofftest(g) → false
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {
    α = max(10, minValue(s, 10, +∞));
    if (α ≥ +∞) break;
}
return α;
```

```
minValue(g7, 10, +∞)
```

```
cutofftest(g7) → false
for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {
    β = min(+∞, 10);
    if (β ≤ 10) break;
}
return β;
```

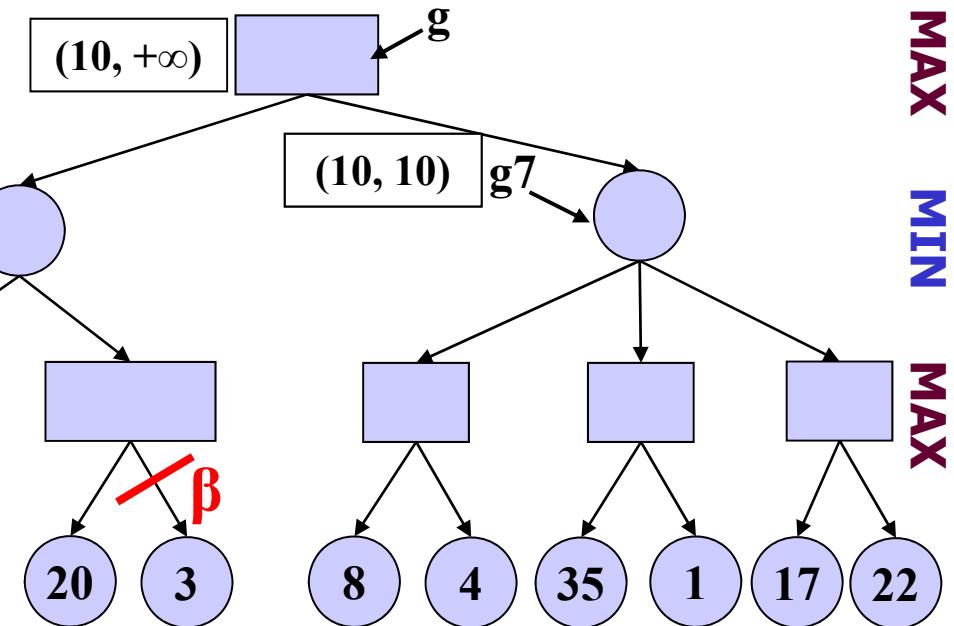


```
maxValue(g, -∞, +∞)
```

```
cutofftest(g) → false
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {
    α = max(10, minValue(s, 10, +∞));
    if (α ≥ +∞) break;
}
return α;
```

```
minValue(g7, 10, +∞)
```

```
cutofftest(g7) → false
for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {
    β = 10;
    if (β ≤ 10) break;
}
return β;
```



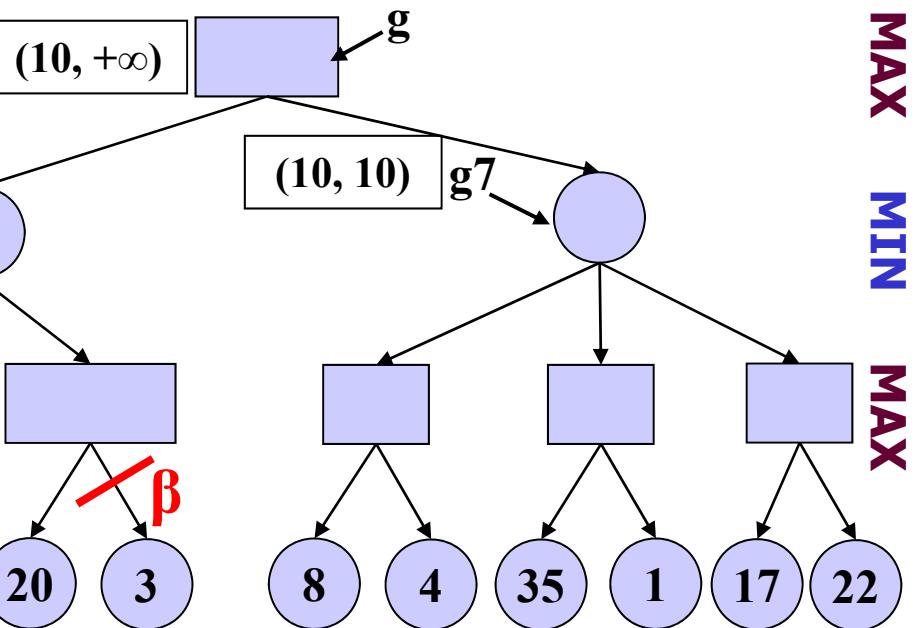
maxValue(g, -∞, +∞)

```
cutofftest(g) → false
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {
    α = max(10, minValue(s, 10, +∞));
    if (α ≥ +∞) break;
}
return α;
```

minValue(g7, 10, +∞)

```
cutofftest(g7) → false
for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {
    β = 10;
    if (10 ≤ β) break;
}
return β;
```

true!

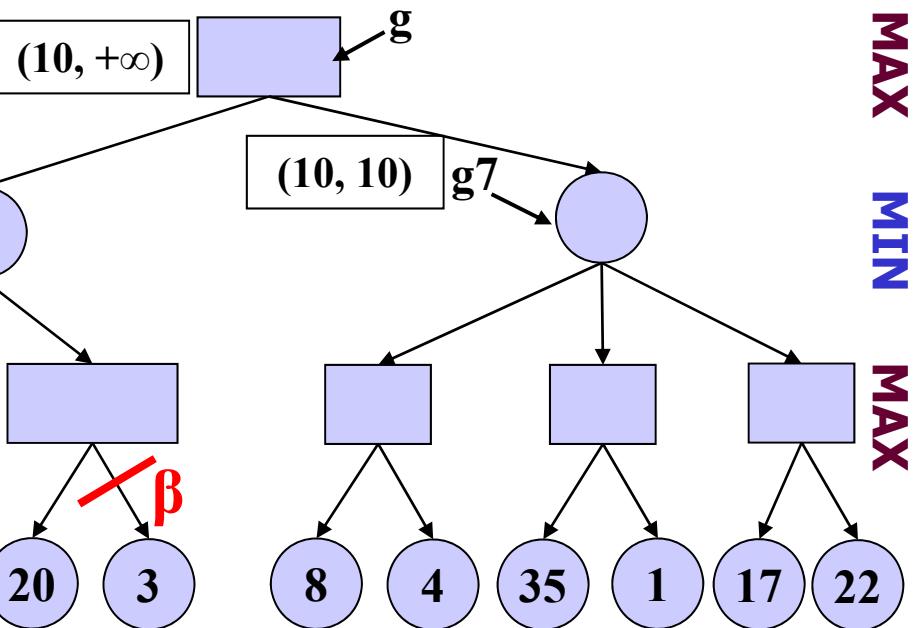


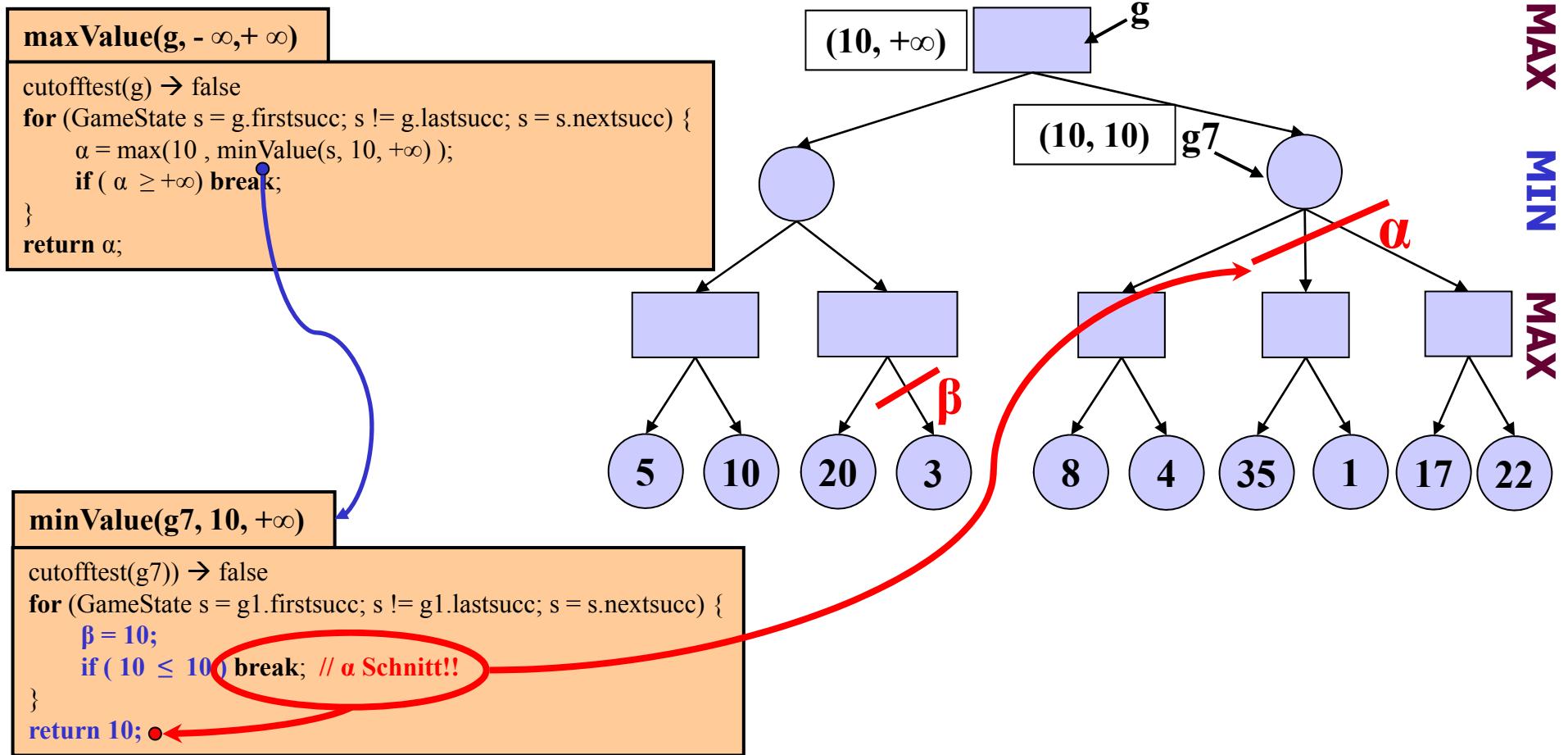
maxValue(g, -∞, +∞)

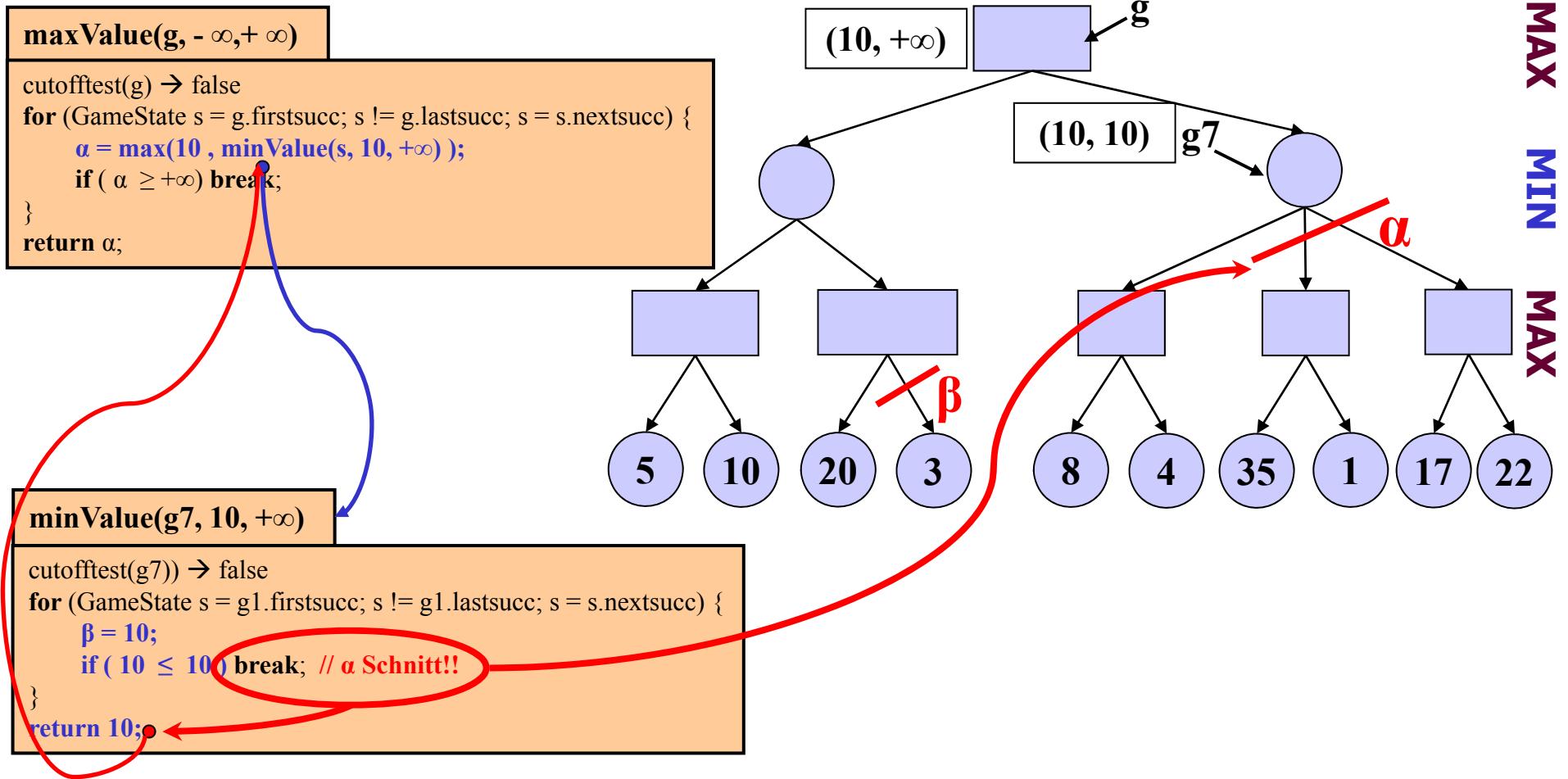
```
cutofftest(g) → false
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {
    α = max(10, minValue(s, 10, +∞));
    if (α ≥ +∞) break;
}
return α;
```

minValue(g7, 10, +∞)

```
cutofftest(g7) → false
for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {
    β = 10;
    if (10 ≤ β) break;
}
return β;
```





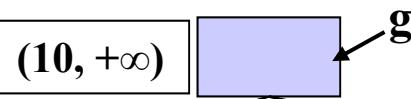


maxValue(g, -∞, +∞)

```
cutofftest(g) → false
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {
    α = max(10, 10);
    if (α ≥ +∞) break;
}
return α;
```

minValue(g7, 10, +∞)

```
cutofftest(g7) → false
for (GameState s = g1.firstsucc; s != g1.lastsucc; s = s.nextsucc) {
    β = 10;
    if (10 ≤ 10) break; // α Schnitt!!
}
return 10;
```



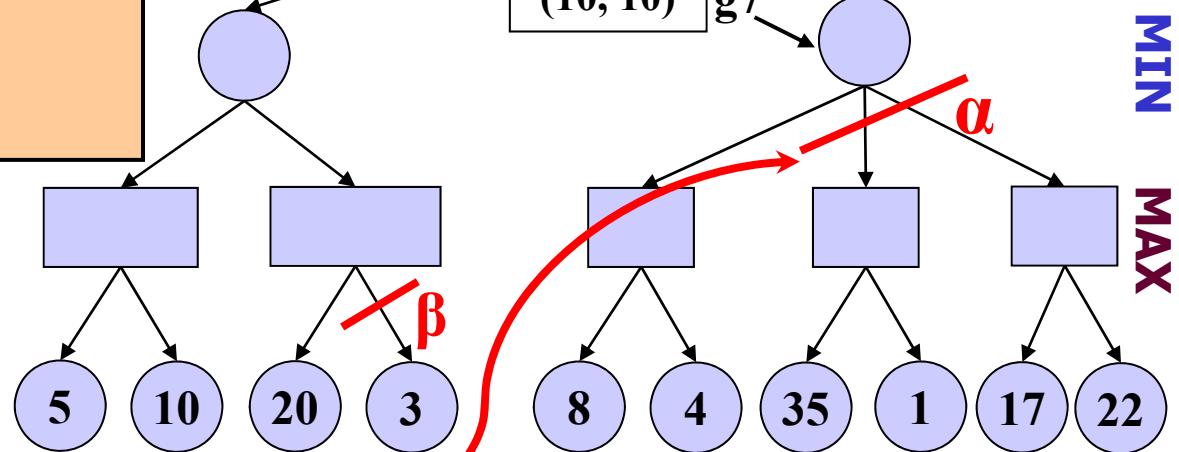
$(10, 10)$

$g7$

MAX

MIN

MAX

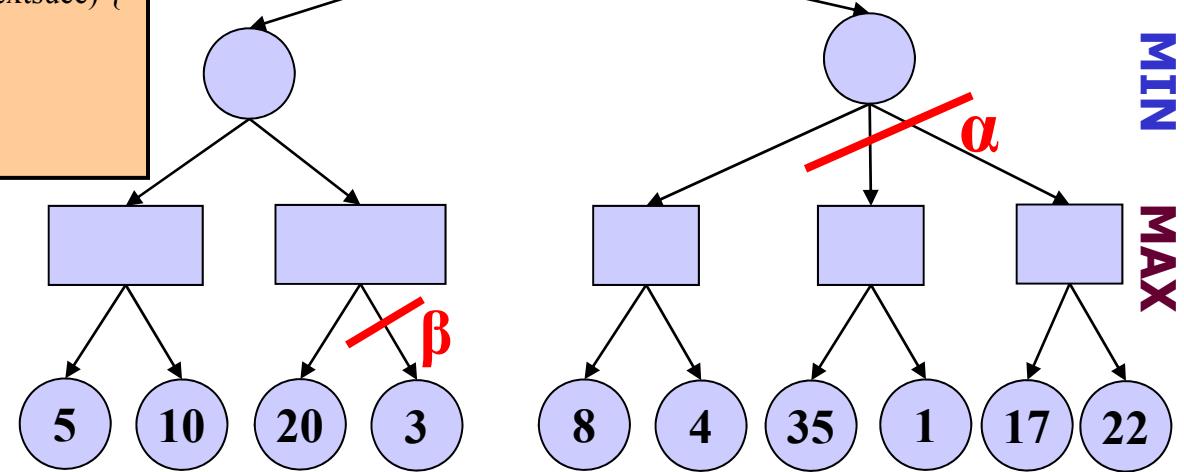


```
maxValue(g, -∞, +∞)
```

```
cutofftest(g) → false  
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {  
    α = 10  
    if (α ≥ +∞) break;  
}  
return α;
```

(10, 10) g

MAX MIN MAX

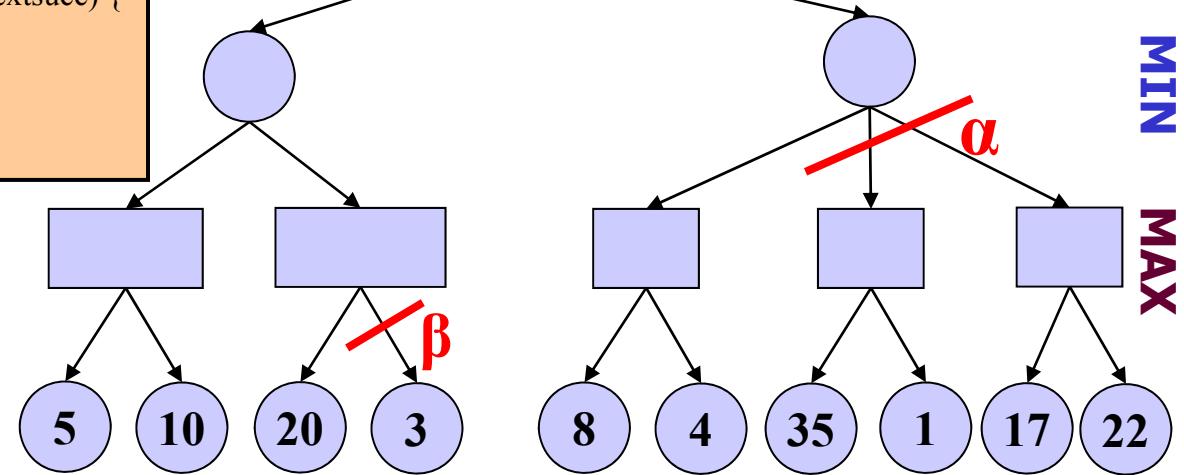


maxValue(g, -∞, +∞)

```
cutofftest(g) → false  
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {  
    a = 10;  
    if ( 10 ≥ +∞) break;  
}  
return a; false!
```

(10, 10) \rightarrow g

MAX
MIN
MAX

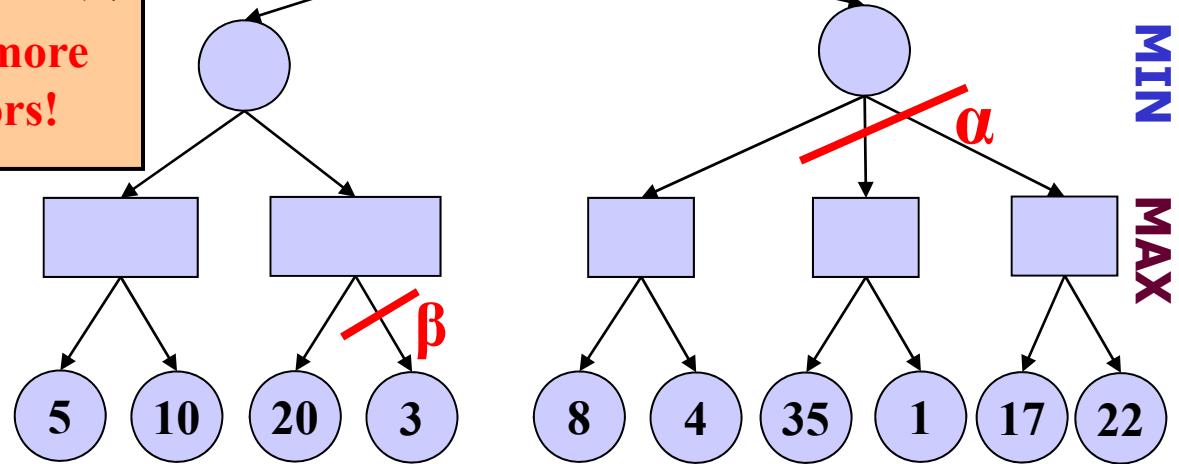


```
maxValue(g, -∞, +∞)
```

```
cutofftest(g) → false  
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {  
    α = 10;  
    if (10 ≥ +∞) break;  
}  
return 10;
```

g has no more successors!

(10, 10) g



```
maxValue(g, -∞, +∞)
```

```
cutofftest(g) → false  
for (GameState s = g.firstsucc; s != g.lastsucc; s = s.nextsucc) {  
    α = 10;  
    if (10 ≥ +∞) break;  
}  
return 10;
```

Maximum yield
from current
position is 10

(10, 10) \rightarrow g

